Homework Solutions #1 -- Physics 6A

Chapter 1

17. **INTERPRET** For this problem, we are looking for an angle subtended by a circular arc, so we will use the definition of angle in radians (see Fig. 1.2).

DEVELOP From Fig. 1.2, we see that the angle in radians is the circular arc length *s* divided by the radius *r*, or $\theta = s/r$.

EVALUATE Inserting the known quantities (s = 2.1 km, *r* = 3.4 km), we find the angle subtended is

$$\theta = \frac{s}{r} = \frac{2.1\text{ km}}{3.4\text{ km}} = 0.62\text{ rad}$$

Using the fact that $\pi\text{ rad} = 180°$, the result can be expressed as

$$\theta = 0.62\text{ rad} = (0.62\text{ rad})\left(\frac{180°}{\pi\text{ rad}}\right) \approx 35°$$

ASSESS Because a complete circular revolution is $360°$, $35°$ is roughly 1/10 of a circle. The circumference of a circle of radius $r = 3.4$ km is $C = 2\pi(3.4\text{ km}) = 21.4\text{ km}$. Therefore, we expect the jetliner to fly approximately 1/10 of *C*, or 2.1 km, which agrees with the problem statement.

20. **INTERPRET** This problem involves calculating the number of seconds in a year, and comparing the result to $\pi \times 10^7$ s.

DEVELOP From Appendix C we find that 1 y = 365.24 d, and that 1 d = 86,400 s.

EVALUATE Using the conversion equations above, we find that one year is

$$1\text{ y} = (1\text{ y})\left(\frac{365.24\text{ d}}{1\text{ y}}\right)\left(\frac{86400\text{ s}}{1\text{ d}}\right) = 3.16\times10^7\text{ s}$$

The percent difference *e* between this result and $\pi \times 10^7$ s is

$$e = \frac{(3.16\times10^7\text{ s} - 3.14\times10^7\text{ s})}{3.16\times10^7\text{ s}}(100\%) = 0.446\%$$

where the result is given to 3 significant figures to match the 3 significant figures of the input (86,400 s/d).

ASSESS To calculate the percent error, we take the difference between the correct result and the erroneous result and divide this difference by the correct result.

39. **INTERPRET** This problem calls for a rough estimate. The quantities of interest here are the total rate of energy consumption (i.e., power consumption) and the area needed for solar cells. The electrical power consumed by the entire population of the United States, divided by the power converted by one square meter of solar cells, is the area required by this question.

DEVELOP For problems that involve rough estimates, various assumptions typically need to be made. Such assumptions must be physically motivated with reasonable order-of-magnitude estimates. Note that we will be required to convert all data regarding power to kW, and that we must express length in common units.

EVALUATE Assume there are 300 million people in the United States, and that the power consumption for each person is 3 kW (a per capita average over 24 h periods of all types of weather), then the total power consumption P_{tot} is

$$P_{\text{tot}} = 300\times10^6 \times 3\ \text{kW} = 9\times10^8\ \text{kW}$$

For a solar cell with 20% efficiency in converting sunlight to electrical power, the power-yield P_{sc} per unit area A is $P_{\text{sc}}/A = (0.20)(300\ \cancel{\text{W}}/\text{m}^2)(10^{-3}\ \text{kW}/\cancel{\text{W}}) = 0.060\ \text{kW/m}^2$. Therefore, the total area A_{tot} needed is

$$A_{\text{tot}} = \frac{P_{\text{tot}}}{P_{\text{sc}}/A} = \frac{9\times10^8\ \text{kW}}{0.060\ \text{kW/m}^2} = 1.5\times10^{10}\ \text{m}^2$$

The land area of the continental United States A_{US} can be approximated as the area of a rectangle the size of the distance from New York to Los Angeles by the distance from New York to Miami,
or $A_{\text{US}} \approx (5000\ \text{km})\times(2000\ \text{km}) = 10^7\ \text{km}^2$. From Table 1.1, we know that 1 km = 10^3 m, or 1 = 10^{-3} km/m. Then the fraction of area to be covered by solar cells would be

$$\frac{A_{\text{tot}}}{A_{\text{US}}} \approx \left(\frac{1.5\times10^{10}\ \cancel{\text{m}^2}}{10^7\ \cancel{\text{km}^2}}\right)\overbrace{\left(\frac{10^{-3}\ \cancel{\text{km}}}{1\ \cancel{\text{m}}}\right)^2}^{=1} = 1.5\times10^{-3}$$

or approximately 0.15%.

ASSESS This represents only a small fraction of land to be used for solar cells. The area A_{tot} is comparable to the fraction of land now covered by airports.

46. **INTERPRET** After giving us several astronomical parameters, this problem asks us to calculate the size of the Sun relative to the Moon and the absolute size of the Sun.

DEVELOP Because the Moon just covers the Sun during a solar eclipse, the Moon and the Sun must subtend the same angle when viewed from the Earth. From Fig. 1.2 we know that $\theta = s/r$, where s is the subtended arc and r is the radius. For $r \gg s$, the arc subtended may be approximated by a straight line, and so may be taken as the diameter of the Moon and the Sun for this problem. Thus, we have $\theta = d_S/r_S = d_M/r_M$, where the subscripts S and M refer to the Sun and the Moon, respectively.

EVALUATE Inserting the given distances to the Sun and the Moon, the ratio of their diameters is

$$\frac{d_S}{r_S} = \frac{d_M}{r_M}$$
$$\frac{d_S}{d_M} = \frac{r_S}{r_M} = \frac{1.5\times10^8\ \text{km}}{4\times10^5\ \text{km}} = 375$$

Rounding this result to a single significant figure gives 400 as the final result. Thus, the Sun has about 400 times the diameter of the Moon. If the Moon's radius is 1800 km, then the Sun's radius is

$$d_S = d_M\frac{r_S}{r_M} = (1800\ \text{km})(375) = 6.75\times10^5\ \text{km}$$

Rounding this result to a single significant figure gives 7×10^5 km as the final radius of the Sun.

ASSESS Notice that we retained more significant figures than warranted when we entered the result of the intermediate calculation into the final calculation. However, at the end of each calculation, we rounded the result to the correct number of significant figures. The angle subtended by each object is (using data for the Moon)

$$\theta = d_M/r_M = (3600\ \text{km})/(4\times10^5\ \text{km}) = (4.5\times10^{-3}\ \text{rad})(180°/\pi\ \text{rad}) = 0.5°$$

47. **INTERPRET** This is a problem that calls for a rough estimate, instead of a precise numerical answer. The quantities of interest here are the size of the electronic components on a PC chip, and the number of calculations that can be performed each second.

DEVELOP The area of each component is the area of the chip divided by the number of components. We'll take the square root of that to get the component size $d_{\text{comp}} = \sqrt{A_{\text{comp}}}$. For part (b), to estimate the number of calculations performed per second, we take the inverse of how long it takes to do one calculation. We are told that one calculation requires that an electric impulse traverse 10^4 components each one million times. The time it takes to traverse one component is the distance across one component divided by the velocity, $t_{\text{comp}} = d_{\text{comp}}/v$. We assume the velocity is approximately the speed of light.

EVALUATE (a) The area of each component is

$$A_{\text{comp}} = \frac{A_{\text{chip}}}{N_{\text{comp}}} = \frac{(4\ \text{mm})^2}{10^9} = 1.6\times10^{-14}\,\text{m}^2$$

Assuming the component is square

$$d_{\text{comp}} = \sqrt{A_{\text{comp}}} = \sqrt{1.6\times10^{-14}\text{m}^2} = 1.3\times10^{-7}\,\text{m} \sim 0.1\ \mu\text{m}$$

(b) The time to do one calculation is the number of components to be traversed, multiplied by the number of traversals, and then multiplied by the time to traverse one component

$$t_{\text{calc}} = (10^4)(10^6)t_{\text{comp}} = 10^{10}\frac{d_{\text{comp}}}{v} = 10^{10}\frac{1.3\times10^{-7}\,\text{m}}{3\times10^8\,\text{m/s}} = 4.2\times10^{-6}\,\text{s}$$

This means the chip can do 2×10^5 calculations per second.

ASSESS The number of calculations per second is often referred to as FLOPS (Floating Point Operations Per Second). The performance of the above chip is 200,000 FLOPS. Modern supercomputers use parallel computing to perform at the level of more than a trillion FLOPS (or TFLOPS).

55. **INTERPRET** We have to adjust the cost per bag of coffee to include the shipping costs.

DEVELOP To find the final cost per bag, calculate the total cost for the 6 bags of coffee and then divide by 6.

EVALUATE The total purchase is 6 bags plus shipping

$$6(\$8.95)+\$6.90=\$60.60$$

Dividing by the number of bags gives $10.10 per bag.

ASSESS If the same shipping costs applied to one bag of coffee, then the price per bag would be $15.85. So sometimes it pays to buy in bulk.

56. **INTERPRET** This problem involves converting worldwide energy use into the average individual's power consumption.

DEVELOP The energy used in a year is a quantity with the dimensions of power (energy/time). So all we have to do is convert this to the more common power unit of the watt ($1\text{ W}=1\text{ J/s}$). To get the average per capita energy consumption, we will divide by the world population, which is currently around 6.8 billion.

EVALUATE The worldwide energy consumption in watts is

$$450\text{ EJ/y}=\frac{\left(450\times10^{18}\text{ J}\right)}{\left(\pi\times10^{7}\text{ s}\right)}=1.4\times10^{13}\text{ W}$$

We have used the approximation for converting years to seconds. Dividing this by the number of people gives

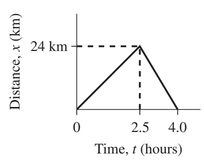
$$\frac{1.4\times10^{13}\text{ W}}{6.8\times10^{9}}\approx 2\text{ kW}$$

ASSESS The United States is said to account for 25% of the world's energy consumption (105 EJ), while accounting for about 5% of the world's population (0.3 billion). The U.S. per capita consumption is 11 kW, which is the highest in the world.

Chapter 2

14. **INTERPRET** This is a one-dimensional kinematics problem that involves calculating your displacement and average velocity as a function of time. There are two different parts to the problem: in the first part we travel north and in the second part where we travel south.

DEVELOP It will help to plot our displacement as a function of time (see figure below). We are given three points: the point where we start (t, y) = (0 h, 0 km), the point where we stop after traveling north at (t, y) = (2.5 h, 24 km), and the point where we return home at (t, y) = (4 h, 0 km). We can use Equation 2.1, $\bar{v}=\Delta x/\Delta t$, to calculate the average velocity. To calculate the displacement we will subtract the initial position from the final position.



EVALUATE (**a**) After the first 2.5 hours, you have traveled north 24 km, so your change in position (i.e., your displacement) is $\Delta x=x-x_0=24\text{ km}-0\text{ km}=24\text{ km}$, where the x_0 is the initial position and x is the final position.

(**b**) The time it took for this segment of the trip is $\Delta t=t-t_0=2.5\text{ h}-0\text{ h}=2.5\text{ h}$. Inserting these quantities into Equation 2.1, we find the average velocity for this segment of the trip is

$$\bar{v}=\frac{\Delta x}{\Delta t}=\frac{24\text{ km}}{2.5\text{ h}}=9.6\text{ km/h}$$

(**c**) For the homeward leg of the trip, $\Delta x=x-x_0=0\text{ km}-24\text{ km}=-24\text{ km}$, and $\Delta t=t-t_0=4.0\text{ h}-2.5\text{ h}=1.5\text{ h}$, so your average velocity is

$$\bar{v}=\frac{\Delta x}{\Delta t}=\frac{-24\text{ km}}{1.5\text{ h}}=-16\text{ km/h}.$$

(**d**) The displacement for the entire trip is $\Delta x=x-x_0=0\text{ km}-0\text{ km}=0\text{ km}$, because you finished at the same position as you started.

(**e**) For the entire trip, the displacement is 0 km, and the time is 4.0 h, so the average velocity is

$$\bar{v}=\frac{\Delta x}{\Delta t}=\frac{0\text{ km}}{1.5\text{ h}}=0\text{ km/h}$$

ASSESS We see that the average velocity for parts (b) and (c) differ in sign, which is because we are traveling in the opposite direction during these segments of the trip. Also, because we return to our starting point, the average velocity for the entire trip is zero—we would have finished at the same position had we not moved at all!

15. **INTERPRET** This problem asks for the time it will take a light signal to reach us from the edge of our solar system.

DEVELOP The time is just the distance divided by the speed: $\Delta t=\Delta x/v$. The speed of light is $3.00\times10^{8}\text{ m/s}$ (recall Section 1.2).

EVALUATE Using the above equation

$$\Delta t=\frac{\Delta x}{v}=\frac{\left(14\times10^{9}\text{ mi}\right)}{\left(3.00\times10^{8}\text{ m/s}\right)}\left(\frac{1609\text{ m}}{1\text{ mi}}\right)=7.5\times10^{4}\text{ s}=21\text{ h}$$

ASSESS It takes light from the Sun 8.3 minutes to reach Earth. This means that the Voyager spacecraft will be 150 times further from us than the Sun.

26. **INTERPRET** For this problem, we need to calculate the time it takes for the airplane to reach its take off speed given its acceleration. Notice that this is similar to the previous problems, except that we are given the velocity and acceleration and are solving for the time, whereas before we were given the velocity and time and solved for acceleration.

DEVELOP We can use Equation 2.4, $\bar{a} = \Delta v/\Delta t$, to solve this problem. We can assume the airplane's initial velocity is $v_1 = 0$ km/h, and we are given the final velocity ($v_2 = 320$ km/h), so the change in the airplane's velocity is $\Delta v = v_2 - v_1 = 320$ km/h. The average acceleration is given as $\bar{a} = 2.9\ \text{m/s}^2$. Notice that the velocity and the acceleration are given in different units, so we will convert km/h to m/s for the calculation.

EVALUATE Insert the known quantities into Equation 2.4 and solve for the time interval, Δt. This gives

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

$$\Delta t = \frac{\Delta v}{\bar{a}} = \left(\frac{320\ \text{km/h}}{2.9\ \text{m/s}^2}\right)\left(\frac{10^3\ \text{m}}{\text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 31\ \text{s}$$

ASSESS With an average acceleration of 2.9 m/s2, the airplane's velocity increases by just under 3 m/s each second. Given that 320 km/h is just under 90 m/s, the answer seems reasonable because if you increment the velocity by 3 m/s 30 times, it will attain 90 m/s.

34. **INTERPRET** The electrons are accelerated to high-speed beforehand. We are only asked to consider the rapid deceleration that occurs when they slam into the tungsten target.

DEVELOP We are given the initial and final velocities, as well as the time duration of the deceleration. We are not asked what the deceleration is, but merely what distance the electrons penetrate the tungsten before stopping. Equation 2.9 is therefore what we will use.

EVALUATE Plugging in the given values we find the stopping distance is

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t = \tfrac{1}{2}(10^8\ \text{m/s} + 0)(10^{-9}\text{s}) = 0.05\ \text{m}$$

ASSESS The electrons are initially travelling close to the speed of light, but only a thin sheet of tungsten is needed to stop them. The X rays that are produced in this way are called *bremsstrahlung,* which means "braking radiation."

37. **INTERPRET** This problem involves constant acceleration due to gravity. We are asked to calculate the distance traveled by the rock before it hit the water.

DEVELOP We chose a coordinate system where the positive-x axis is downward. We are given the rock's constant acceleration (gravity, $g = 9.8\ \text{m/s}^2$), its initial velocity $v_0 = 0.0$ m/s, and its travel time t = 4.4 s. Insert this data into Equation 2.10 and solve for the displacement $x - x_0$.

EVALUATE From Equation 2.10, we find

$$x - x_0 = v_0 t + \frac{1}{2}at^2 = v_0 t + \frac{1}{2}gt^2$$

$$= (0.0\ \text{m/s})(4.4\ \text{s}) + \frac{1}{2}(9.8\ \text{m/s}^2)(4.4\ \text{s})^2 = 95\ \text{m}$$

ASSESS When the travel time of the sound is ignored, the depth of the well is quadratic in t. The depth of the well is about the length of an American football field. If we use the speed of sound $s = 340$ m/s, how will that change our answer?

58. **INTERPRET** This is a one-dimensional kinematics problem with constant deceleration. We are given the final velocity, the acceleration distance, and the acceleration distance, and we are asked to find the initial velocity and the acceleration time.

DEVELOP We choose a coordinate system in which the positive-x direction is in the direction of the car's initial velocity. Using the known quantities (v = 18 kh/h, a = $-6.3\ \text{m/s}^2$, $x - x_0 = 34$ m), solve Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$, for the initial velocity v_0. Then use the result for v_0 in Equation 2.7, $v = v_0 + at$, to find the acceleration time t. Converting the final velocity to m/s for the calculation, we have v = (18 km/h)(1 h/3600 s)(10^3 m/km) = 5.0 m/s.

EVALUATE **(a)** Inserting the known quantities into Equation 2.11 gives

$$v_0 = \sqrt{v^2 - 2a(x - x_0)} = \sqrt{(5.0\ \text{m/s})^2 - 2(-6.3\ \text{m/s}^2)(34\ \text{m})} = 21\ \text{m/s}$$

(b) Inserting this result for v_0 into Equation 2.7 gives

$$t = \frac{v - v_0}{a} = \frac{5.0\ \text{m/s} - 21.3\ \text{m/s}}{-6.3\ \text{m/s}^2} = 2.6\ \text{s}$$

where we have retained more significant figures for v_0 because it serves as an intermediate result for this part.

ASSESS In km/h, the initial velocity is v_0 = (21.3 m/s)(10^{-3} km/m)(3600 s/h) = 77 km/h.

65. **INTERPRET** This is a one-dimensional kinematics problem that involves finding the vertical distance of an object as a function of time.

DEVELOP Choose a coordinate system in which the positive-x direction is upward. Equation 2.10, $x(t) = x_0 + v_0 t + at^2/2$, describes the vertical position $x(t)$ of an object falling from x_0 as a function of time. Because the object was dropped from a stationary position, $v_0 = 0$ so $x(t) = x_0 + at^2/2$. Furthermore, we are free to choose the origin of the x axis where we like, so we let $x_0 = 0$, which gives $x(t) = at^2/2$.
Finally, the acceleration is $a = -g = -9.8\ \text{m/s}^2$, which points downward, so our Equation 2.10 takes the form $x(t) = -gt^2/2$. The problem states that $x(t) - x(t-1) = x(t)/4$, from which we can solve for t, which we can insert into $x(t)$ to find x (i.e., the height from which it was dropped). Notice that x will be negative because the object's final position is below its initial position.

EVALUATE

$$x(t) - x(t-1) = \frac{x(t)}{4}$$
$$-\frac{1}{2}gt^2 - \left[-\frac{1}{2}g(t-1)^2\right] = -\frac{1}{8}gt^2$$
$$\frac{1}{2}g(1-2t) = -\frac{1}{8}gt^2$$
$$t^2 - 8t + 4 = 0$$
$$t = 4 \pm 2\sqrt{3}\ \text{m/s}$$

(We discarded the negative square root because $t > 1$ s.) Inserting this result into $x(t)$ gives

$$x(t) = -\frac{1}{2}gt^2 = -\frac{1}{2}(9.8\ \text{m/s}^2)\left[\left(4+2\sqrt{3}\right)\text{s}\right]^2 = -270\ \text{m}$$

to two significant figures. Thus, the object must be dropped from a height of 273 m.

ASSESS During a free fall, the vertical distance traveled is proportional to t^2. Therefore, we expect the object to travel a greater distance during the latter time interval. In general, we must also take into consideration air resistance.

84. **INTERPRET** This problem involves calculating the time it takes a water balloon to reach the ground.

DEVELOP We know from previous problems that the balloon will reach the ground in a time of $t = \sqrt{2h/g}$, where h is the height from which it is released. In order for the balloon to hit its target, the distance, d, between the X and the impact point must be vt, where v here is the typical velocity of students entering the building.

EVALUATE Putting together the information above

$$d = vt = v\sqrt{\frac{2h}{g}} = (2\ \text{m/s})\sqrt{\frac{2(20\ \text{m})}{(9.8\ \text{m/s}^2)}} = 4\ \text{m}$$

ASSESS Since not all the students will be walking at the average 2 m/s, a more effective strategy would be to use 2 X's on the ground farther out from the building. By measuring the time it takes a given student to walk between the X's, you can measure his/her speed. From that, you can more accurately predict when they will be underneath your window, and you will therefore know for sure when to release your balloon.

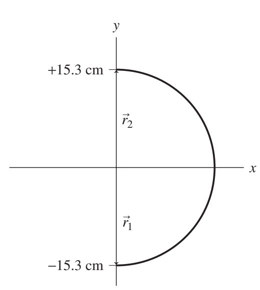
Chapter 3

12. **INTERPRET** This problem involves finding the distance of a semicircular arc given its radius, then finding the magnitude the resulting displacement if one travels along this arc.

DEVELOP The length of an arc is the radius multiplied by the angle in radians, or $s = r\theta$ (see Figure 1.2). For a semicircular arc, $\theta = \pi$ radians.

EVALUATE **(a)** The length of the semicircle is $s = \pi r = \pi(15.2\ \text{cm}) = 47.8\ \text{cm}$ **(b)** The magnitude of the displacement vector, from the start of the semicircle to its end, is just the diameter of the circle, which is twice the radius. Therefore, $d = 2r = 2(15.2\ \text{cm}) = 30.4\ \text{cm}$.

ASSESS The displacement may also be found by subtracting the final position vector $\vec{r}_2$ from the initial position vector $\vec{r}_1$ (see figure below). This gives $|\vec{r}_2 - \vec{r}_1| = 15.2\ \text{cm} + 15.2\ \text{cm} = 30.4\ \text{cm}$.



13. **INTERPRET** This problem involves the addition of two displacement vectors in two dimensions and finding the magnitude and direction of the resultant vector.

DEVELOP Using Equation 3.1, we see that in two dimensions, a vector $\vec{A}$ can be written in unit vector notation as

$$\vec{A} = A_x\hat{i} + A_y\hat{j} = A\left[\cos(\theta_A)\hat{i} + \sin(\theta_A)\hat{j}\right]$$

where $A = \sqrt{A_x^2 + A_y^2}$ and $\theta_A = \text{atan}\left(A_y/A_x\right)$. Similarly, we express a second vector $\vec{B}$ as $\vec{B} = B_x\hat{i} + B_y\hat{j} = B\left[\cos(\theta_B)\hat{i} + \sin(\theta_B)\hat{j}\right]$. The resultant vector $\vec{C}$ is

$$\vec{C} = \vec{A} + \vec{B} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j} = \left[A\cos(\theta_A) + B\cos(\theta_B)\right]\hat{i} + \left[A\sin(\theta_A) + B\sin(\theta_B)\right]\hat{j} = C_x\hat{i} + C_y\hat{j}$$

EVALUATE From the problem statement, $A = 360$ km and $\theta_A = 135°$ (see figure below). The first segment of travel can thus be written as

$$\vec{A} = A\left(\cos\theta_A\hat{i} + \sin\theta_A\hat{j}\right) = (360\text{ km})\left[\cos(135°)\hat{i} + \sin(135°)\hat{j}\right] = (-254.6\text{ km})\hat{i} + (254.6\text{ km})\hat{j}$$

Similarly, the second segment of the travel can be expressed as (with $B = 400$ km and $\theta_B = 90°$)

$$\vec{B} = B\left[\cos(\theta_B)\hat{i} + \sin(\theta_B)\hat{j}\right] = (400\text{ km})\hat{j}$$

Thus, the resultant displacement vector is

$$\vec{C} = \vec{A} + \vec{B} = C_x\hat{i} + C_y\hat{j} = (-254.6\text{ km})\hat{i} + (254.6\text{ km} + 400\text{ km})\hat{j} = (-254.6\text{ km})\hat{i} + (654.6\text{ km})\hat{j}$$

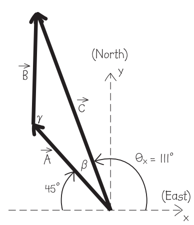
The magnitude of $\vec{C}$ is

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-254.6\text{ km})^2 + (654.6\text{ km})^2} = 702.4\text{ km} \approx 700\text{ km}$$

to two significant figures. Its direction is

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \text{atan}\left(\frac{654.6\text{ km}}{-254.6\text{ km}}\right) = -68.75°,\text{ or }111°$$

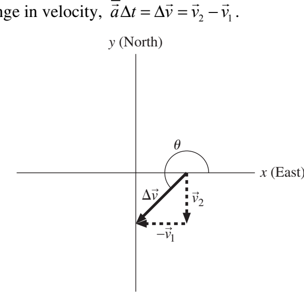
We choose the latter solution (110° to two significant figures) because the vector (with $C_x < 0$ and $C_y > 0$) lies in the second quadrant.



ASSESS As depicted in the figure, the resultant displacement vector $\vec{C}$ lies in the second quadrant. The direction of $\vec{C}$ can be specified as 111° CCW from the *x*-axis (east), or $45° + 23.7° = 68.7°$N of W.

22. **INTERPRET** This problem involves calculating an average acceleration vector given its initial and final velocity.

DEVELOP Draw a diagram of the situation (see figure below) to display graphically the difference $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$, where v_1 is the intial velocity and v_2 is the final velocity. From Equation 3.5, we know that the average acceleation is in the same direction as the change in velocity, $\bar{\vec{a}}\Delta t = \Delta\vec{v} = \vec{v}_2 - \vec{v}_1$.

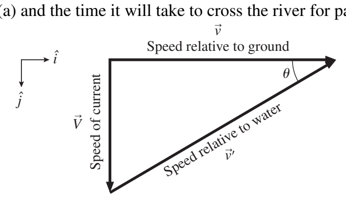


EVALUATE The angle made by $\Delta\vec{v}$ is $\theta = \text{atan}\left(\frac{v_2}{v_1}\right) = \text{atan}(-1) = 225°$, where we have used $|\vec{v}_1| = |\vec{v}_2|$, which we know because the initial and final speed is the same. Thus, the average acceleration is oriented at 225° counter-clockwise from the *x* axis.

ASSESS The direction may also be reported as 135° clockwise from the *x* axis.

27. **INTERPRET** This problem involves relative motion. You are asked to find the direction of the velocity with respect to the water so that a boat traverses the current perpendicularly with respect to the shore. You also need to find the time it takes to cross the river.

DEVELOP Choose a coordinate system where $\hat{i}$ is the direction perpendicular to the water current, and $\hat{j}$ is the direction of the water current (see figure below). The velocity of the current $\vec{V}$ relative to the ground is $\vec{V} = (0.57 \text{ m/s})\hat{j}$, and the magnitude of the velocity $\vec{v}'$ of the boat relative to the water is $v' = 1.3$ m/s. We also known that the velocity $\vec{v}$ of the boat relative to the shore is in the $\hat{i}$ direction, so $\vec{v} = v\hat{i}$, and that the river is d = 63 m wide. These three vectors satisfy Equation 3.7, $\vec{v} = \vec{v}' + \vec{V}$, as drawn in the figure below. This allows us to find the direction of $\vec{v}'$ for part (a) and the time it will take to cross the river for part (b).



EVALUATE **(a)** From the figure above, we see that

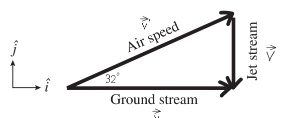
$$\sin(\theta) = V/v'$$
$$\theta = \text{asin}(V/v') = \text{asin}(0.57 \text{ m/s}/1.3 \text{ m/s}) = 26^\circ$$

(b) To find the time to traverse the river, we calculate the speed v with respect to the shore. From the figure above, we see that $v = v'\cos(\theta) = (1.3 \text{ m/s}) \cos 26° = 1.17$ m/s, so the crossing time t is $t = d/v = (63 \text{ m})/(1.17 \text{ m/s}) = 53.9 \text{ s} = 54 \text{ s}$ to two significant figures.

ASSESS In this time, you will have rowed a distance d' relative to the water of $d' = v't = (1.3 \text{ m/s})(53.9 \text{ s}) = 70$ m.

28. **INTERPRET** This problem involves relative velocities. We are asked to calculate the speed of the jet stream given the velocity of an airplane relative to the ground and the direction at which the airplane must fly relative to the ground to perpendicularly traverse the jet stream.

DEVELOP Use Equation 3.7, $\vec{v} = \vec{v}' + \vec{V}$ to find $\vec{V}$, the velocity of the jet stream. This relationship is shown graphically in the figure below. Here, $\vec{v}$ is the velocity of the airplane relative to the ground, and $v' = 370$ km/h is the speed of the airplane relative to the air. Notice that the angle between $\vec{v}'$ and $\vec{v}$ is 32°, as given in the problem statement.



EVALUATE From trigonometry, the magnitude of the jet stream speed is $V = v'\sin(\theta) = (370 \text{ km/h})\sin(32°) = 196$ km/h.

ASSESS The speed of the airplane relative to the ground is $v = v'\cos(\theta) = (370 \text{ km/h})\cos(32°) = 314$ km/h. The plane's heading of 32° north of east is a reasonable compensation for the southward wind blowing at a speed of 196 km/h.

50. **INTERPRET** This is a problem of relative velocities. The ferryboat has to head upstream slightly to compensate for the current that drags it downstream.

DEVELOP The river velocity $\vec{V}$ and the ferryboat velocity $\vec{v}'$ with respect to the water are two sides of a right triangle (see the figure in the solution to Exercise 3.27), where the third side is the velocity of the boat relative to the ground.

EVALUATE (a) The angle that the boat must head is given by

$$\theta = \sin^{-1}\left(\frac{V}{v'}\right)$$

(b) If V were greater than v', then there would be no solution for the angle (since $|\sin\theta| \le 1$). What this means is that the river is flowing too fast for the ferryboat to be able to get straight across.

ASSESS If $V = v'$, there's no real solution, since that would mean $\theta = 90°$. At that heading (straight upstream), the ferryboat would not be moving towards the other side of the river. It would be motionless relative to the ground.