Homework #2 Solutions

Chapter 3

31. **Interpret** This problem asks you to determine how far your sailboard goes during a gust of wind that results in a constant acceleration.

Develop We'll assume the initial velocity is in the positive x direction ($\vec{v}_0 = 6.5\hat{i}$ m/s). The acceleration can be broken up into x and y components as follows:

$$a_x = a\cos\theta = \left(0.48\ \text{m/s}^2\right)\cos 35^\circ = 0.393\ \text{m/s}^2$$

$$a_y = a\sin\theta = \left(0.48\ \text{m/s}^2\right)\sin 35^\circ = 0.275\ \text{m/s}^2$$

To find the displacement, we can use Equation 2.10 for both the x and y directions.

Evaluate The displacement in the x direction is

$$\Delta x = v_0 t + \tfrac{1}{2}a_x t^2 = (6.5\ \text{m/s})(6.3\ \text{s}) + \tfrac{1}{2}\left(0.393\ \text{m/s}^2\right)(6.3\ \text{s})^2 = 48.7\ \text{m}$$

The displacement in the y direction is

$$\Delta y = \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}\left(0.275\ \text{m/s}^2\right)(6.3\ \text{s})^2 = 5.46\ \text{m}$$

The magnitude and direction of the displacement are

$$\Delta r = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(48.7\ \text{m})^2 + (5.46\ \text{m})^2} = 49\ \text{m}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{5.46\ \text{m}}{48.7\ \text{m}}\right) = 6.4^\circ$$

Assess The angle of the displacement is less than that of the acceleration. That makes sense because the initial velocity was along the x axis, and therefore there should be a greater displacement in that direction.

34. **Interpret** This problem involves an object moving under the influence of gravity near the Earth's surface, so it is projectile motion. We are asked to find the horizontal distance traveled by an arrow given its initial horizontal velocity, vertical velocity, and the height from which it is shot.

Develop The horizontal and vertical motions of the arrows are independent of each other, so we can consider them separately. The time of flight t of the arrow can be determined from its range (horizontal motion, Equation 3.12). Once t is found, we can insert it into the equation of motion for the vertical direction (Equation 3.13) to determine the initial height.

Evaluate From Equation 3.12, the total flight time of the arrow is

$$t = \frac{x - x_0}{v_{0x}} = \frac{23\ \text{m}}{41\ \text{m/s}} = 0.561\ \text{s}$$

Substituting this result into Equation 3.13, and noting that $v_{y0} = 0$, the height from which the arrow was shot is

$$y_0 = y + \frac{1}{2}gt^2 = 0.0\ \text{m} + \frac{1}{2}\left(9.8\ \text{m/s}^2\right)(0.561\ \text{s})^2 = 1.5\ \text{m}$$

to two significant figures.

Assess Dropping a height of 1.5 m in half a second is reasonable for free fall. We may relate y_0 to x as

$$y_0 = \frac{1}{2}gt^2 = \frac{1}{2}g\left(\frac{x - x_0}{v_{0x}}\right)^2$$

From which it is clear that the larger is y_0, the longer it takes for the arrow to reach the ground, and the greater the horizontal distance traveled.

39. **Interpret** This problem asks us to estimate the acceleration of the Moon given its orbital radius and its orbital period. Because the Moon's orbit is nearly circular, we can use the formulas for uniform circular motion.

Develop For uniform circular motion, the centripetal (i.e., center-seeking) acceleration is given by Equation 3.16, $a = v^2/r$, where v is the orbital speed and r is the orbital radius. The problem states that $r = 3.85 \times 105$ km and that the orbital period T is $T = 27$ days = 648 h. The orbital speed is the distance covered in one period divided by the period, or $v = 2\pi r/T$.

Evaluate Inserting the given quantities into Equation 3.16, we find

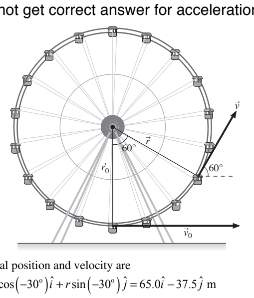
$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2\left(3.85\times 10^5\ \text{km}\right)}{(648\ \text{h})^2} = \left(36\ \text{km/h}^2\right)\left(\frac{10^6\ \text{mm}}{\text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)^2 = 2.8\ \text{mm/s}^2$$

Assess The direction of the acceleration is always towards the center of the Earth.

49. **Interpret** In this problem we have to find the average velocity and acceleration by taking the difference in the position vector and the velocity vector, and then dividing by the time.

Develop Let's choose a coordinate system with origin at the center of the Ferris wheel, so that the position vector always has a magnitude of $r = \tfrac{1}{2}150\ \text{m} = 75\ \text{m}$. The speed is the circumference divided by the rotational period: $v = 2\pi \cdot 75\text{m}/30\text{min} = 0.262\ \text{m/s}$. Let's take the initial position to be at the lowest point, i.e., $\vec{r}_0 = -75\hat{j}$ m , and we'll assume the wheel moves counterclockwise, such that $\vec{v}_0 = 0.262\hat{i}$ m/s . After $\Delta t = 5.0$ min , the wheel will have completed 1/6th of its rotation, meaning it will have advanced by 60° . The final position will be -30° from the x direction, while the final velocity will be 60° from the x direction. See the figure below.

Note: full credit if you did not get correct answer for acceleration.



In component form, the final position and velocity are

$$\vec{r} = r\cos(-30^\circ)\hat{i} + r\sin(-30^\circ)\hat{j} = 65.0\hat{i} - 37.5\hat{j}\ \text{m}$$

$$\vec{v} = v\cos(60^\circ)\hat{i} + v\sin(60^\circ)\hat{j} = 0.131\hat{i} + 0.227\hat{j}\ \text{m/s}$$

Evaluate (a) The average velocity is change in position divided by the time:

$$\bar{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\left(65.0\hat{i} - 37.5\hat{j}\ \text{m}\right) - \left(-75\hat{j}\ \text{m}\right)}{5.0\ \text{min}} = 0.22\hat{i} + 0.13\hat{j}\ \text{m/s}$$

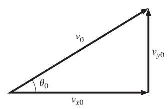
(b) The average acceleration is change in velocity divided by the time:

$$\bar{\vec{a}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\left(0.131\hat{i} + 0.227\hat{j}\ \text{m/s}\right) - \left(0.262\hat{i}\ \text{m/s}\right)}{5.0\ \text{min}} = \left(-4.4\hat{i} + 7.6\hat{j}\right)\times 10^{-4}\,\text{m/s}^2$$

Assess The magnitude of the average velocity is 0.25 m/s, which is nearly the same as the instantaneous velocity of 0.26 m/s. The average velocity is smaller because it doesn't take into account the curved path followed by a point on the rim of the wheel.

64. **INTERPRET** This problem involves projectile motion. We are asked to express the maximum horizontal range in terms of the angle at which a projectile is launched and the maximum height it attains.

DEVELOP The expression for the horizontal range (when the initial and final heights are equal) is $x = 2v_0^2 \sin(\theta_0)$ (Equation 3.15). The maximum height $h = y_{\max} - y_0$ can be found from Equation 2.11, $v_y^2 = v_{y0}^2 - 2g(y_{\max} - y_0)$, with $v = 0$, which gives $v_{y0} = \sqrt{2gh}$. If you draw a picture of the initial velocity vector and its components (see figure below), it becomes apparent that $\cos(\theta_0) = v_x/v_0$, $\sin(\theta_0) = v_y/v_0$, and $\tan(\theta_0) = v_{y0}/v_{x0}$.



Therefore, from the equation just before Equation 3.15, we have $x = (2v_0^2/g)\sin(\theta_0)\cos(\theta_0) = 2v_{x0}v_{y0}/g$. Combine these equations to solve the problem.

EVALUATE Inserting $\tan(\theta_0) = v_{y0}/v_{x0}$ and $v_{y0} = \sqrt{2gh}$ into the last expression from above for x gives

$$x = \frac{2v_{x0}v_{y0}}{g} = \frac{2v_0^2}{g\tan(\theta_0)} = \frac{4h}{\tan(\theta_0)}$$

ASSESS This result reflects a classical geometrical property of the parabola, namely, that the latus rectum is four times the distance from vertex to focus.

65. **INTERPRET** This problem involves projectile motion. You are asked to estimate the initial horizontal speed of the motorcyclist given the range over which he flew.

DEVELOP Imagine the motorcyclist is traveling at the legal speed, 60 km/h = 16.67 m/s. If we find that his range is less than the 39 m reported, we can conclude that he was probably not speeding. If his range is greater than 39 m, then he was probably speeding. Assume that he is deflected upwards off the car's windshield (which we consider to be a frictionless surface), at 45°, which will maximize his range. We can then use Equation 3.15 to find the range over which he would travel before landing on the road.

EVALUATE Inserting the intial speed and angle into Equation 3.15 gives

$$x = \frac{v_0^2}{g}\sin(2\theta_0) = \frac{(16.67\text{ m/s})^2 \sin(90^\circ)}{9.8\text{ m/s}^2} = 28\text{ m}$$

Because of our assumptions, this would be the motorcyclist's maximum range. The fact that he flew 39 m before landing implies that he was almost certainly speeding.

ASSESS To estimate the minimum speed at which he was traveling, insert the range of x = 39 m into Equation 3.15 and solve for the initial velocity v_0 (again assuming $\theta_0 = 45°$). This gives

$$x = \frac{v_0^2}{g}\sin(2\theta_0)$$

$$v_0 = \pm\sqrt{\frac{xg}{\sin(2\theta_0)}} = \pm\sqrt{\frac{(39\text{ m})(9.8\text{ m/s}^2)}{\sin(90^\circ)}} = (19.56\text{ m/s})\left(\frac{1\text{ km}}{10^3\text{ m}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 70\text{ km/h}$$

78. **INTERPRET** This problem asks you to find the initial angle, θ_0, that gives the maximum range, x, for the trebuchet.

DEVELOP The general case of a projectile launched with speed v_0 from a height h is tackled in Problem 3.79. As this is a rather complicated derivation, we will not reproduce it here, but instead use the result:

$$\theta_{\max} = \tfrac{1}{2}\cos^{-1}\left(\frac{1}{1 + v_0^2/gh}\right)$$

EVALUATE Plugging in the launching speed and height of the cliff:

$$\theta_{\max} = \tfrac{1}{2}\cos^{-1}\left(\frac{1}{1 + (36\text{ m/s})^2/(9.8\text{ m/s}^2)(75\text{ m})}\right) = 34^\circ$$

ASSESS We can plug $\theta_0 = \theta_{\max}$ and $y = -h$ into Equation 3.14:

$$(-75\text{ m}) = x\tan 34^\circ - \frac{(9.8\text{ m/s}^2)}{2(36\text{ m/s})^2\cos^2 34^\circ}x^2$$

Using the quadratic formula, we find a range of $x \approx 190$ m . If we instead had chosen $\theta_0 = 45^\circ$, the range would have been slightly smaller, $x \approx 180$ m .

Note: full credit for attempting #78.

79. **INTERPRET** This problem asks you to find the initial angle, θ_0, that gives the maximum range, x, for a projectile launched with speed v_0 from a height h . Recall that the maximum occurs when the derivate, $dx/d\theta_0$, is zero.

DEVELOP We need to find an equation that relates x and θ_0 . Let's assume the projectile is launched from the origin, so that it lands at a vertical position of $y = -h$. We can find the range from Equation 3.14,

$$y = -h = x\tan\theta_0 - \frac{g}{2v_0^2\cos^2\theta_0}x^2$$

Let's rearrange this equation by multiplying through by $\cos^2\theta_0$ and defining $H = v_0^2/2g$ (which is the maximum height of the stone's trajectory using Equation 2.11)

$$x^2 - 4xH\sin\theta_0\cos\theta_0 - 4hH\cos^2\theta_0 = 0$$

Now using the trigonometric identities: $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 2\cos^2\theta - 1$, we have

$$x^2 - 2xH\sin 2\theta_0 - 2hH\left(\cos 2\theta_0 + 1\right) = 0$$

We could solve for x using the quadratic formula, but that will get messy. Instead, we will leave the equation like this and take the derivative with respect to θ_0. We can then set $dx/d\theta_0$ equal to zero and then solve for the angle that gives the maximum range.

Evaluate In taking the derivative of the above equation, we are careful to apply the chain rule and product rule from Appendix A:

$$2x\cdot\frac{dx}{d\theta_0} - 2H\left[\frac{dx}{d\theta_0}\cdot\sin 2\theta_0 + 2x\cos 2\theta_0\right] - 2hH\left[-2\sin 2\theta_0\right] = 0$$

If we then assume $dx/d\theta_0 = 0$ for the maximum range, we are left with

$$-4Hx_{\max}\cos 2\theta_{\max} - 4hH\sin 2\theta_{\max} = 0 \;\rightarrow\; x_{\max} = h\tan\theta_{\max}$$

where $\theta_{\max}$ is the angle that gives the maximum range, $x_{\max}$. Notice that $\theta_{\max} = 45^\circ$ is undefined except for $h = 0$, which would be the normal case of a trajectory over level ground (see Equation 3.75). To solve for $\theta_{\max}$ generally, we plug it and the expression for $x_{\max}$ into the trajectory equation that we derived above:

$$h^2\tan^2 2\theta_{\max} - 2hH\tan 2\theta_{\max}\sin 2\theta_{\max} - 2hH\left(\cos 2\theta_{\max} + 1\right) = 0$$
$$h\sin^2 2\theta_{\max} - 2H\sin^2 2\theta_{\max}\cos 2\theta_{\max} - 2H\cos^2 2\theta_{\max}\left(\cos 2\theta_{\max} + 1\right) = 0$$

Using the fact that $\sin^2\alpha = 1 - \cos^2\alpha = \left(1 - \cos\alpha\right)\left(1 + \cos\alpha\right)$, the above equation reduces to:

$$\cos 2\theta_{\max} = \frac{1}{1 + 2H/h}$$

Or equivalently

$$\theta_{\max} = \tfrac{1}{2}\cos^{-1}\left(\frac{1}{1 + v_0^2/gh}\right)$$

Assess If we assume the ground is level ($h = 0$), then the argument in the $\cos^{-1}$ function goes to zero, which means $\theta_{\max} = 45^\circ$, as it should when the trajectory is over level ground.

Chapter 4

15. **Interpret** This problem involves Newton's second law. The object of interest is the passenger, and we are to calculate the force required to stop the passenger in the given time.

Develop Assume that the seatbelt holds the passenger firmly to the seat, so that the passenger also stops in 0.14 s without incurring any secondary impact. The passenger's average acceleration is $a_{av} = (0 - v_0)/t$ and his mass is 60 kg. Insert these quantities into Newton's second law to find the force.
Evaluate The average force exerted by the seatbelt on the passenger is

$$F_{av} = ma_{av} = -mv_0/t = -\frac{(60\text{ kg})}{0.14\text{ s}}(110\text{ km/h})\left(\frac{1000\text{ m}}{\text{km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = -13\text{ kN}$$

Assess The negative sign indicates that the force is opposite to the direction of the initial velocity.

17. **Interpret** This problem involves Newton's 2^{nd} law for constant mass.
Develop By Equation 4.3, the kinesin force imparts an acceleration on the molecular complex of $a = F/m$.
Evaluate Recall from Appendix B that the SI prefix pico (p) corresponds to 10^{-12}, so

$$a = \frac{F}{m} = \frac{6.0\times10^{-12}\text{ N}}{3.0\times10^{-18}\text{ kg}} = 2.0\times10^{6}\text{ m/s}^2$$

Assess This is an extraordinarily large acceleration, but it would only be applied for a fraction of a second, so the final velocity would be reasonable.

23. **Interpret** This problem asks us to find the mass of an object whose weight on the Moon corresponds to the weight of 35-kg object on the Earth.
Develop Use Equation 4.5 to find the weight of the block on the Earth. Use the gravitational acceleration g_M from Appendix E to calculate the mass that corresponds to an object of this weight on the Moon.

Evaluate To lift a 35-kg block on Earth requires a force at least equivalent to its weight, which is $w = mg = (35\text{ kg})(9.8\text{ m/s}^2) = 343\text{ N}$. The same force on the moon could lift a mass $m = w/g_M = (343\text{ N})/(1.62\text{ m/s}^2) = 212\text{ kg} \approx 210\text{ kg}$ to two significant figures.

Assess The weight of a 212-kg object on Earth is $w = mg = (212\text{ kg})(9.8\text{ m/s}^2) = 2078\text{ N}$, which is a factor g/g_M = $(9.8\text{ m/s}^2)/(1.62\text{ m/s}^2)$ = 6 times more than the weight on the Moon. Thus, you can lift 6 times the mass on the Moon than you can on the Earth.

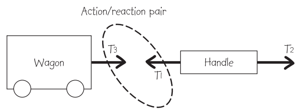
32. **Interpret** This problem involves kinematics (to find the acceleration of the person), Newton's second law (to find forces acting on the person), and Newton's third law. The forces involved are the gravitational force and the normal force exerted by the floor of the elevator on the person's feet (see free-body diagram from Problem 4.29).
Develop Because this is a one-dimensional problem, we can dispense with the vector notation, provided we assign positive values to upward vectors and negative values to downward vectors. The average acceleration is (see Equation 3.5) $\bar{a} = \Delta v/\Delta t = (-9.3\text{ m/s}^2)/(2.1\text{ s}) = -4.38\text{ m/s}^2$. The apparent weight w_{ap} is simply the force you exert

37. **Interpret** This is a one-dimensional problem that involves Hooke's law and Newton's third law. We are asked to find the distance a spring with a given spring constant is stretched if we apply a given force to it.
Develop We apply the same reasoning as per Problem 4.36, except that we choose a coordinate system in which the applied force is in the negative-x direction. The problem states that k = 340 N/m and the applied force is the gravitational force (Equation 4.5) on the fish: $F_{app} = w = mg = -(6.7\text{ kg})(9.8\text{ m/s}^2)$.
Evaluate Inserting the given quantities into Hooke's law gives

$$x = -\frac{F_{sp}}{k} = \frac{F_{app}}{k} = \frac{-(6.7\text{ N})(9.8\text{ m/s}^2)}{340\text{ N/m}} = -0.19\text{ m} = -19\text{ cm}$$

Thus the spring stretches 19 cm downward.
Assess Notice that the spring is extended in the negative-x direction, as expected if we apply a force in that direction.

46. **Interpret** This problem asks us to consider the tension in the handle when the handle and the wagon are accelerated. The key concepts involved here are Newton's second and third laws.
Develop There are two forces on the handle: the tension from the wagon resisting the motion (we'll call this $\vec{T}_1$) and the tension from the child's pulling (we'll call this $\vec{T}_2$). See the figure below.



We'll assume that the only force on the wagon is from the tension in the handle, which we have denoted as $\vec{T}_3$. Using the second law, the net horizontal force on the handle and wagon are, respectively,

$$F_{\text{net,h}} = T_2 - T_1 = m_h a$$
$$F_{\text{net,w}} = T_3 = m_w a$$

Where we have assumed that the positive direction is to the right. Since by the third law, $\vec{T}_1$ and $\vec{T}_3$ are an action/reaction pair, $T_1 = m_w a$. Plugging this in above, we have $T_2 = (m_w + m_h)a$.

Evaluate Solving for the tension on both sides of the handle

$$T_1 = (11\text{ kg})(2.3\text{ m/s}^2) = 25\text{ N}$$
$$T_2 = (11\text{ kg} + 1.8\text{ kg})(2.3\text{ m/s}^2) = 29\text{ N}$$

These tensions are not equal because if they were, the net force on the handle would be zero and it wouldn't accelerate (contrary to what we are told). One can also argue that the T_3 - T_1 pair is less than T_2 because the former has only has to accelerate the wagon, whereas latter has to accelerate both the wagon and the handle.

Assess Often times physics problems involving a string (or some other force-transferring object) will assume for simplicity that the string is massless. Under such an approximation, the tensions on the two ends of the string will be equal, since the net force on a massless object is always zero.

51. **Interpret** The problem asks us to determine the crumple zone of a car, in order to keep the stopping force on a passenger below a given value.

Develop We can think of the crumple zone as the distance, Δx, the car and its passengers continue to travel as they go from the initial speed to zero. We can use Equation 2.11 to relate this distance to the deceleration of the car,

$$v^2 = 0 = v_0^2 - 2a\Delta x$$

Note that we have included a negative sign, so that a is a positive quantity. Using Equation 4.3, we can derive a limit on the crumple zone from the requirement that the force on the passenger must be less than 20 times his/her weight:

$$F \le 20F_g \quad \rightarrow \quad a \le 20g$$

Evaluate The crumple zone is the distance during the crash over which the car comes to rest, so $\Delta x = v_0^2/2a$. Using the limit on the acceleration, the crumple zone must be at least

$$\Delta x = \frac{v_0^2}{2a} \ge \frac{v_0^2}{2(20g)} = \frac{(70\text{ km/h})^2}{40(9.8\text{ m/s}^2)} = 0.96\text{ m}$$

Assess This says the car would have to crumple by almost a meter. That's quite a bit, but the pictures of cars in high-speed collisions seem to imply that modern cars can compress by this much.

52. **Interpret** This is an application of Newton's 2nd law.

Develop We're given the acceleration of the frog tongue and its mass, so the force needed is just $F = ma$.

Evaluate Plugging in the given values

$$F = ma = \left(500\times10^{-6}\,\mathbf{kg}\right)\left(250\ \mathbf{m/s^2}\right) = 0.13\ \mathbf{N}$$

Assess This is a reasonable amount of force to expect from a frog. The acceleration is so large because the frog's tongue has such a small mass.

56. **Interpret** You are asked to find out how many passengers an elevator can accommodate within the guideline of safety standards. The forces involved here are the downward gravitational force $\vec{F}_g$ and the upward cable tension $\vec{T}$.

Develop Assume that the only forces involved are $\vec{F}_g$ and $\vec{T}$ in the vertical direction. Newton's second law gives $\vec{F}_{\text{net}} = \vec{T} + \vec{F}_g = M\vec{a}$, where M is the total mass of the elevator and its passengers. Taking $+y$ to point upward, the equation in component form is $T - Mg = Ma_y$, which implies the total mass is

$$T = M(g + a_y)$$

The tension is greatest when the elevator is accelerating upward ($a_y > 0$).

Evaluate For safety's sake, we require that

$$T \le \tfrac{2}{3}T_{\max} = \frac{2}{3}(19.5\text{ kN}) = 13.0\text{ kN}$$

Assuming the elevator is accelerating upward at its maximum rate ($a_y = 2.24\text{ m/s}^2$), the total mass is limited t

$$M = \frac{T}{g + a_y} \le \frac{13.0\text{ kN}}{9.8\text{ m/s}^2 + 2.24\text{ m/s}^2} = 1080\text{ kg}$$

Subtracting the mass of the elevator (490 kg), the maximum load in terms of kg and 70-kg passengers is:

$$\text{Max load} = 1080\text{ kg} - 490\text{ kg} = 590\text{ kg}$$
$$= 590\text{ kg}\left(\frac{\text{person}}{70\text{ kg}}\right) = 8\text{ persons}$$

Assess An elevator that accommodates 8 passengers, with a total mass of 590 kg sounds reasonable. Many passenger elevators, depending on their size, can accommodate up to about 2500 kg.

67. **Interpret** We're asked to calculate the amount of jerk on an amusement ride, where jerk is the rate of change in acceleration.

Develop The word "rate" implies per time. The jerk is the time derivative of the acceleration. We're given an equation for the force, so the acceleration is just this divided by the mass, M, of the car and passengers.

Evaluate The acceleration on the amusement ride is

$$a = \frac{F}{M} = \frac{F_0}{M}\sin\omega t$$

The jerk is the time derivative of this:

$$\frac{da}{dt} = \frac{F_0}{M}\omega\cos\omega t$$

The maximum value of the cosine is 1, so the maximum jerk is equal to $\omega F_0/M$.

Assess If the maximum jerk is too high, some of the passengers may suffer a whiplash.

Chapter 5

14. **Interpret** In this problem, we are asked to find the tilt angle of an air table such that the acceleration of an object sliding on the surface of the table is the same as the gravitational acceleration near the surface of the Moon.

Develop Example 5.1 shows that the acceleration down an incline is $a = g\sin(\theta)$. By setting the acceleration equal to the acceleration due to gravity on the surface of the Moon ($a = g_M = 1.6\text{ m/s}^2$), we can solve for the tilt angle θ.

EVALUATE The angle of tilt should be

$$\theta = \operatorname{asin}\left(\frac{a}{g}\right) = \operatorname{asin}\left(\frac{1.6\ \text{m/s}^2}{9.8\ \text{m/s}^2}\right) = 9.4°$$

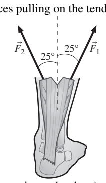
above the horizontal.

ASSESS Notice that the tilt angle does not depend on the mass of the object.

Note: Problem 17 is done incorrectly. The sec on part of the problem should use cosine, rather than sine. Full credit for attempting.

17. **INTERPRET** This is a static problem in which we are looking for the force exerted on the tendon by the two muscles.

DEVELOP In this case, there are two forces pulling on the tendon as shown below in the figure:



We are told that the horizontal pulls are opposite each other (meaning that the two forces are in the x-y plane), and we assume that the net horizontal force is zero: $F_1 \sin 25° - F_2 \sin 25° = 0$, in which case $F_1 = F_2$. The net vertical force pulls up on the tendon with a force equivalent to ten times the gymnast's weight:

$$F_1 \sin 25° + F_2 \sin 25° = 10mg$$

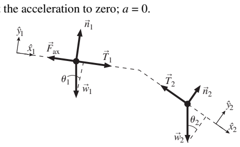
EVALUATE Solving for the force in each muscle gives

$$F_1 = F_2 = \frac{10(55\ \text{kg})(9.8\ \text{m/s}^2)}{2\sin 25°} = 6.4\ \text{kN}$$

ASSESS The Achilles tendon is the thickest and strongest tendon in the body. In simply walking, it has to withstand strains of as much as 4 times the body weight.

21. **INTERPRET** This is a two-dimensional problem that involves applying Newton's second law to two climbers, tied together by a rope and sliding down an icy mountainside. The physical quantities of interest are their acceleration and the force required to bring them to a complete stop.

DEVELOP We choose two coordinate systems where the x axes are parallel to the slopes and y axes are perpendicular to the slopes (see figure below). Assume that the icy surface is frictionless and that the climbers move together as a unit with the same magnitude of down-slope acceleration a. If the rope is not stretching, the tension forces are equal in magnitude, so $T_1 = T_2 \equiv T$. To find the acceleration of the climbers, apply Newton's second law in the direction of the slope. To find the force F_{ax} exerted by the ax, again apply Newton's second law, but this time include F_{ax} and set the acceleration to zero; $a = 0$.



EVALUATE **(b)** For this part, we neglect the force due to the ax. Because we are now working in one-dimension (the x dimension), we forego vector notation, and insert the sign (±) according to the direction of the force. Of course, at the end we must interpret the sign of the resulting force as indicating its direction (positive or negative x direction). The magnitude of the net force in the $\hat{x}_1$ and $\hat{x}_2$ directions (downward positive) is thus

$$\begin{aligned} F_{net} &= m_1 g\sin(\theta_1) + \overbrace{T_1 - T_2}^{=0} + m_2 g \sin(\theta_2) \\ &= (75\ \text{kg})(9.8\ \text{m/s}^2)\sin(12°) + (63\ \text{kg})(9.8\ \text{m/s}^2)\sin(38°) = 533\ \text{N} \end{aligned}$$

Thus, the magnitude of the acceleration of the pair is

$$a = \frac{F_{net}}{m_1 + m_2} = \frac{533\ \text{N}}{75\ \text{kg} + 63\ \text{kg}} = 3.9\ \text{m/s}^2$$

so the pair accelerate down the slope at 3.9 m/s^2.

(b) After they have stopped, we include the force of the ax. Thus, the magnitude of the force due to the ax is

$$\begin{aligned} F_{net} &= -F_{ax} + m_1 g \sin(\theta_1) + \overbrace{T_1 - T_2}^{=0} + m_2 g \sin(\theta_2) = m \overbrace{a}^{=0} = 0 \\ F_{ax} &= m_1 g \sin(\theta_1) + m_2 g \sin(\theta_2) = 530\ \text{N} \end{aligned}$$

so two significant figures. That is, the force exerted by the ax must be 530 N up the slope.

ASSESS If the two climbers were not roped together, then their acceleration would have been

$$a_1 = \frac{F_1}{m_1} = g\sin(\theta_1) = (9.8\ \text{m/s}^2)\sin(12°) = 2.04\ \text{m/s}^2$$

$$a_2 = \frac{F_2}{m_2} = g\sin(\theta_2) = (9.8\ \text{m/s}^2)\sin(38°) = 6.03\ \text{m/s}^2$$

The acceleration of the pair is the mass-weighted average of the individual accelerations:

$$\begin{aligned} a &= \frac{F_{net}}{m_1+m_2} = \frac{m_1 g\sin(\theta_1) + m_2 g\sin(\theta_2)}{m_1+m_2} = \left(\frac{m_1}{m_1+m_2}\right) g\sin(\theta_1) + \left(\frac{m_2}{m_1+m_2}\right) g \sin(\theta_2) \\ &= \left(\frac{m_1}{m_1+m_2}\right) a_1 + \left(\frac{m_2}{m_1+m_2}\right) a_2 = 3.9\ \text{m/s}^2 \end{aligned}$$

24. **INTERPRET** For the rock to whirl around in a circle, the string has to supply the centripetal force through its tension. To keep this tension below the limit, the string makes an angle with the horizontal, as shown in Figure 5.11 in the text.

DEVELOP The situation is the same as described in Example 5.5. The net vertical force is zero, so the weight is balanced by the vertical component of the tension $T\sin\theta = mg$. As for the horizontal component of the tension, it is providing the needed centripetal force, $T\cos\theta = mv^2/r$. The radius of the rock's trajectory is $r = L\cos\theta$.

EVALUATE (a) We are first asked to find the minimum angle that keeps the tension under the string's breaking limit:

$$\theta_{min} = \sin^{-1}\left(\frac{mg}{T_{max}}\right) = \sin^{-1}\left(\frac{(0.940\ \text{kg})(9.8\ \text{m/s}^2)}{(120\ \text{N})}\right) = 4.40°$$

(b) At this angle, the speed of the rock is

$$v = \sqrt{\frac{T_{max}L}{m}\cos\theta_{min}} = \sqrt{\frac{(120\ \text{N})(1.30\ \text{m})}{(0.940\ \text{kg})}\cos 4.40°} = 12.8\ \text{m/s}$$

ASSESS The stronger the string is, the closer to the horizontal it can whirl the rock around (i.e. $\theta_{min} \to 0$ as T_{max} increases). But to maintain such a trajectory, the velocity has to increase, so that the centripetal acceleration is sufficient.

30. **INTERPRET** This problem involves Newton's second law, the force due to kinetic friction, and kinematics. The object of interest is the skier, and we are asked to find how much longer it would take him to descend a slope with a non-zero coefficient of kinetic friction as compared to if there were no friction.

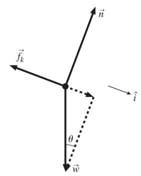
DEVELOP Start with a free-body diagram, and choose a coordinate system in which the positive-x direction is down the slope (see figure below). The forces acting on the skier are the force of gravity, $w = mg$, the normal force n exerted by the slope, and the force f_k due to kinetic friction. To find the skier's acceleration, apply Newton's second law in the $\hat{i}$ direction This gives

$$F_{\text{net}} = ma$$
$$-f_k + w\sin(\theta) = ma$$

where $w = mg$ and we have made $f_k < 0$ because it always acts to oppose the motion, so in this case it acts in the negative-x direction. The force due to kinetic friction may be found from Equation 5.3, $f_k = \mu_k n = \mu_k mg\cos(\theta)$, where we have used Newton's second law in the $\hat{j}$ direction in the final equality:

$$F_{\text{net}} = n - mg\cos(\theta) = ma = 0 \quad \Rightarrow \quad n = mg\cos(\theta)$$

We can now calculate the acceleration a, from which we can find the time to descend the slope using kinematic Equation 2.10 for constant acceleration, $x = x_0 + v_0 t + at^2/2$, with $x - x_0 = 100$ m and $v_0 = 0$ m/s.



EVALUATE If there is no kinetic friction ($\mu_k = 0$ so fk = 0), then, from the first equation above, the acceleration is $a_1 = g\sin(\theta)$. From Equation 2.10, the time to descend the slope is

$$x - x_0 = \overbrace{v_0}^{0} t_1 + \frac{a_1 t_1^2}{2}$$
$$t_1 = \sqrt{\frac{2(x - x_0)}{a_1}} = \sqrt{\frac{2(x - x_0)}{g\sin(\theta)}} = \sqrt{\frac{2(100\text{ m})}{(9.8\text{ m/s}^2)\sin(28^\circ)}} = 6.59\text{ s}$$

where we have taken the positive square root. If $\mu_k = 0.17$, then the acceleration is $a = g\sin(\theta) - g\mu_k\cos(\theta)$, so the time to descend the slope is

$$t_2 = \sqrt{\frac{2(x - x_0)}{a_2}} = \sqrt{\frac{2(x - x_0)}{g\sin(\theta) - g\mu_k\cos(\theta)}} = \sqrt{\frac{2(100\text{ m})}{(9.8\text{ m/s}^2)\left[\sin(28^\circ) - 0.17\cos(28^\circ)\right]}} = 7.99\text{ s}$$

The difference in the time to descend the slopes is $t_2 - t_1 = 7.99\text{ s} - 6.59\text{ s} = 1.4$ s.

ASSESS Notice that the units for the formulas giving the time are seconds. Considering that the fastest runners can cover 100 m in slightly less than 10 s, we see that our skier travels considerably faster than an extremely fast runner.

34. **INTERPRET** We are asked to find the sum of the forces provided by two motor proteins.

DEVELOP The net force on the spindle pole is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$. We're only asked to find the magnitude of this sum, so let's choose a coordinate system that makes our life easy. If the $+x$ axis splits the middle between the two forces, then $\theta = \pm\frac{1}{2}65^\circ = \pm 32.5^\circ$ and the y-components of the forces ($F_y = F\sin\theta$) cancel each other out. Therefore, the net force points completely in the x-direction.

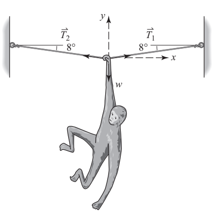
EVALUATE The magnitude is just the sum of the x-components:

$$F_{\text{net}} = F_{1x} + F_{2x} = (7.3\text{ pN})\left[\cos(+32.5^\circ) + \cos(-32.5^\circ)\right] = 12\text{ pN}$$

ASSESS The answer is within the limits of what the sum could be. If the forces were aligned ($\theta = 0$), then the net force would be $2F = 14.6$ N. Whereas if the forces were completely opposite ($\theta = 180^\circ$), then the net force would be zero.

35. **INTERPRET** This problem involves Newton's second law. We are asked to find the tension in a rope needed to support an object of a given mass.

DEVELOP Draw a diagram of the situation (see figure below). Apply Newton's second law in the y direction and solve for the tension of the rope. Note that the tension of the rope is everywhere the same (for a massless rope), so $T_1 = T_2 = T$,



EVALUATE Applied in the y direction, Newton's second law gives

$$T_1\sin(\theta) + T_2\sin(\theta) = w$$
$$T = \frac{mg}{2\sin(\theta)} = \frac{(15\text{ kg})(9.8\text{ m/s}^2)}{2\sin(8^\circ)} = 530\text{ N}$$

The monkey's weight is w = mg = (15 kg)(9.8 m/s2) = 150 N (to two significant figures). This is over three times less than the tension force in the rope.

ASSESS Notice that the $T \to \infty$ as $\theta \to 0$ because there is a vanishingly small component of the tension acting in the vertical direction. The majority of the tension simply serves to pull the two support points together.

38. **INTERPRET** We're asked to calculate the horizontal traction force supplied by a mass and a set of massless/frictionless pulleys.

DEVELOP Because the pulleys are massless and frictionless, the tension T in the cord will be the same throughout the system. This tension has to support the mass from falling, so $T = mg$. The horizontal force on the leg is the sum from the cord above and below the pulley attached to the foot:

$$F_y = T\cos\theta_1 + T\cos\theta_2$$

EVALUATE Using the values given, the traction force on the leg is:

$$F_y = (4.8\text{ kg})(9.8\text{ m/s}^2)(\cos 70^\circ + \cos 20^\circ) = 60\text{ N}$$

ASSESS This force is about 10% of the weight of a 60 kg person, so this seems reasonable for the amount of force needed to counter some of the forces exerted by muscles in the leg.

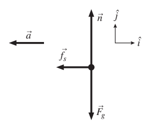
47. **Interpret** This problem involves Newton's second law and kinematics. The object of interest is the train, and we are asked to find if the train can stop within 150 m if the wheels maintain static contact with the rails (i.e., the wheels do not skid on the rails).

Develop Considering all the wheels as one point of contact, make a free-body diagram for the train (see figure below). Applying Newton's law to the train wheels gives $\vec{F}_{\text{net}} = \vec{f}_s + \vec{n} + \vec{F}_g = m\vec{a}$, and writing this in component form gives

$$\begin{aligned} x:&\quad F_g + n = 0 \\ y:&\quad -f_s = -ma \end{aligned}$$

where we have used the fact that there is zero acceleration in the x direction and we have explicitly noted the sign of the friction force and the acceleration to emphasize that they are in the same direction (negative-x direction).

The force due to static friction is $f_s \le \mu_s n$ and the force due to gravity is $F_g = -mg$ (because gravity acts in the downward direction). Insert these values into the above equations to find the maximum acceleration possible without having the wheels slip on the rails, then use the kinematic Equation 2.11 $v^2 = v_0^2 + 2a\left(x - x_0\right)$ to find the stopping distance.



Evaluate Newton's second law thus gives

$$\begin{aligned} ma &= f_s \le \mu_s n = \mu_s mg \\ a &\le \mu_s g \end{aligned}$$

so the maximum acceleration possible is $\mu_s g$. Inserting this result for the acceleration into Equation 2.11 gives a stopping distance of

$$\overbrace{v^2}^{=0} = v_0^2 + 2a\left(x - x_0\right)$$

$$x - x_0 = \frac{v_0^2}{2a} = \frac{v_0^2}{2\mu_s g} = \frac{\left(140\ \text{km/h}\right)^2}{2\left(0.58\right)\left(9.8\ \text{m/s}^2\right)}\left(\frac{10^3\ \text{m}}{\text{km}}\right)^2\left(\frac{\text{h}}{3600\ \text{s}}\right)^2 = 130\ \text{m}$$

so the train will stop before hitting the car.

Assess The stopping time for the train is

$$\begin{aligned} x - x_0 &= \left(v_0 + v\right)t/2 \\ t &= \frac{2\left(x - x_0\right)}{v_0} = \frac{2\left(133\ \text{m}\right)}{38.9\ \text{m/s}} = 6.8\ \text{s} \end{aligned}$$

which should be just enough time for the passengers to get out of the car.

63. **Interpret** This problem involves Newton's second law, uniform circular motion, and frictional forces. The object of interest is the car, and we are to find whether braking in a straight line will stop the car before it hits the truck, or whether it's better to swerve in as tight a circular turn as possible. The forces acting on the car are the force due to gravity $\vec{F}_g = m\vec{g}$ and the force due to kinetic friction $\vec{f}_k$ for the former option and the force due to static friction $\vec{f}_s$ for the latter option.

Develop For the braking option, Newton's second law applied to the car in the x and y directions gives

$$\left.\begin{aligned} x:&\quad f_k = ma \\ y:&\quad n - mg = 0 \end{aligned}\right\} \mu_s g = a\,,$$

where we have used Equation 5.3 $f_s = \mu_s n$. For the swerve option, Newton's second law applied in the x and y directions gives

$$\left.\begin{aligned} x:&\quad f_s = ma = mv^2/r \\ y:&\quad n - mg = 0 \end{aligned}\right\} \mu_s g = v^2/r$$

Use the kinematic Equation 2.11 $v^2 = v_0^2 + 2a\left(x - x_0\right)$ to find the stopping distance in the braking option, and calculate the turning radius r for the swerve option. Compare these results to decide which option to take.

Evaluate For the braking option, the stopping distance is

$$\overbrace{v^2}^{=0} = v_0^2 + 2a\left(x - x_0\right)$$

$$x - x_0 = -\frac{v_0^2}{2a} = -\frac{v_0^2}{2\left(-\mu_s g\right)} = \frac{v_0^2}{2\mu_s g}$$

where the acceleration has a negative sign because it is oriented opposite to the velocity. For the swerving option, the turning radius is $r = v^2/\mu_s g$ = $(x - x_0)$. Thus the turning radius is greater then the stopping distance, so you should chose to brake in a straight line rather than swerve.

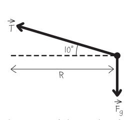
Assess Note that if the coefficient of static friction decreases from its maximum value of μ_s, the turning radius will get larger, and the linear acceleration will decrease, as expected.

72. **Interpret** This problem involves Newton's second law and uniform circular motion. We need to compare the tangential speed of the hammer as it goes around the circle with that of a "speeding bullet." The forces acting on the hammer are the force of gravity $\vec{F}_g = m\vec{g}$, and the tension force from the cable.

Develop Draw a free-body diagram of the hammer as seen from the side (see figure below). Applying Newton's second law in the horizontal and vertical directions gives

$$\left.\begin{aligned} x:&\quad T\cos\left(\theta\right) = ma = mv^2/r \\ y:&\quad T\sin\left(\theta\right) = mg \end{aligned}\right\} \frac{v^2}{r} = \frac{g}{\tan\left(\theta\right)}$$

where we have used Equation 5.1 $a = mv^2/r$ for the centripetal acceleration that the hammer experiences. We can now solve for the speed.



Evaluate Solving the above equation for the tangential speed v gives

$$v^2 =$$

$$v = \pm\sqrt{\frac{rg}{\tan\left(\theta\right)}} = \pm\sqrt{\frac{\left(2.4\ \text{m}\right)\left(9.8\ \text{m/s}^2\right)}{\tan\left(10^\circ\right)}} = \pm 11.5\ \text{m/s}$$

which is an order of magnitude slower than a speeding bullet.

Assess Notice that the units under the radical are m^2/s^2. The positive and negative answers correspond to the hammer turned clockwise and counter clockwise around the circle.