Chapter 6

15. **INTERPRET** This problem involves the average force exerted by the meteorite on the Earth. It is a one-dimensional problem because all forces and displacements are in the same direction (i.e., vertical).

DEVELOP Because we are interested in the average force, which is constant during the meteorite's deceleration period, we can use Equation 6.1 $W = F\Delta x$ to find the average force. We are given the W = 140 MJ and Dx = 75 cm = 0.75 m.

EVALUATE Solving Equation 6.1 for the force and inserting the given quantities gives an average force of

$$W = F\Delta x$$
$$F = \frac{W}{\Delta x} = \frac{140\ \text{MJ}}{0.75\ \text{m}} = 190\ \text{MN}$$

to two significant figures.

ASSESS Notice that we did not need to convert from MJ to J, we simply retained the prefactor M (= 10^6) in our calculation. Thus, the units of MN are units of force. Using the fact that dynamite carries 7.5 MJ/kg of explosive energy, this meteorite impact delivered the equivalent of (140 MJ)/(7.5 MJ/kg) ≈ 19 kg of dynamite (about 41 lbs).

24. **INTERPRET** We're asked how much work does a fly impart on a spider silk strand, assuming the strand acts like a simple spring.

DEVELOP As calculated for Equation 6.10, the work done on a spring when stretching it is: $W = \frac{1}{2}kx^2$.

EVALUATE Using the spring constant of the strand and the distance it stretches when a fly hits it, the work done is

$$W = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(70\ \text{mN/m})(0.096\ \text{m})^2 = 0.32\ \text{mJ}$$

ASSESS When the fly hits the strand, it transfers some significant fraction of its kinetic energy into the work used to stretch the strand. We can estimate the fly's kinetic energy before the impact. Let's assume the fly has a mass of roughly 1 g and that its speed is around 1 m/s. Then, its kinetic energy $\left(K = \frac{1}{2}mv^2\right)$ is 0.5 mJ. So the work we calculated above seems reasonable.

27. **INTERPRET** This problem involves kinetic energy. We are to find the speed at which the small car must travel so that it has the same kinetic energy as the large truck.

DEVELOP We will use Equation 6.13, K =mv2/2, to find the kinetic energy of each vehicle. By setting their kinetic energies equal, we can solve for the speed of the car.

EVALUATE Let the car's variables carry the subscript c, and the truck's variables carry the subscript T. The kinetic energy of each is $K_c = m_c v_c^2/2$ for the car and $K_T = m_T v_T^2/2$ for the truck. Setting these equal and solving for v_c gives

$$\frac{1}{2}m_c v_c^2 = \frac{1}{2}m_T v_T^2$$
$$v_c = \pm v\sqrt{\frac{m_T}{m_c}} = \pm(20\ \text{km/h})\sqrt{\frac{3.2\times 10^4\ \text{kg}}{950\ \text{kg}}} = \pm 120\ \text{km/h}$$

ASSESS The plus/minus sign indicates that the car can either travel in the same direction as the truck, or in the opposite direction. Notice that we did not need to convert km/h to m/s for this problem, because the units of kg under the radical cancel.

31. INTERPRET This problem is an exercise in converting power from kcal/day to Watts.

DEVELOP From Appendix C, we find that 1 cal = 4.184 J, and we know that 1 day = (24 h)(3600 s/h) = 86,400 s.

EVALUATE Performing the conversion gives

$$\frac{2000\ \text{kcal}}{1\ \text{d}}\overbrace{\left(\frac{1\ \text{d}}{86,400\ \text{s}}\right)}^{=1}\overbrace{\left(\frac{1000\ \text{cal}}{1\ \text{kcal}}\right)}^{=1}\overbrace{\left(\frac{1\ \text{J}}{4.184\ \text{cal}}\right)}^{=1}=5.53\ \text{J/s}=5.53\ \text{W}$$

ASSESS This is an *average* power. Human power output is higher during exercise.

36. INTERPRET This problem involves the work-energy theorem and average power. We are asked to find the power output of a long-jumper during his prejump run.

DEVELOP The work-energy theorem (Equation 6.14) states that $\Delta K = W_{\text{net}}$, and from the net work we can calculate the power output using Equation 6.15, $\bar{P} = \Delta W/\Delta t$.

EVALUATE The energy expended in the prejump run is

$$W_{\text{net}} = \frac{m}{2}\left(v_2^2 - \overbrace{v_1^2}^{=0}\right) = \frac{mv_2^2}{2}$$

Therefore, the average power is

$$\bar{P} = \frac{\Delta W}{\Delta t} = \frac{mv_2^2}{2\Delta t} = \frac{(75\ \text{kg})(10\ \text{m/s})^2}{2(3.1\ \text{s})} = 1.2\ \text{k W}$$

to two significant figures.

ASSESS Note that the power output is proportional to the final speed squared.

43. INTERPRET You want to find out how much work you do during a particular exercise.

DEVELOP You only do work when lifting the weight (gravity does the work to bring the weight back down). The work required to lift the weight the given distance is $W = w\Delta y$ (it's irrelevant at what angle the force from your arms is applied – the net result is that the weight moves up by Δy). We'll need to convert the work to kcal using $1\ \text{kcal} = 4184\ \text{J}$ from Appendix C.

EVALUATE (a) Each repetition requires you to exert

$$W = w\Delta y = (20\ \text{N})(0.55\ \text{m}) = 11\ \text{J}\left(\frac{1\ \text{kcal}}{4184\ \text{J}}\right) = 2.63\times10^{-3}\text{kcal}$$

To get a 200 kcal workout, the number of reps you'd have to do is

$$N = \frac{200\ \text{kcal}}{2.63\times10^{-3}\text{kcal}} = 76,000$$

(b) If your workout takes 1.0 min, then the power output is just the work divided by the time:

$$P = \frac{W}{\Delta t} = \frac{200\ \text{kcal}}{1.0\ \text{min}}\left(\frac{4184\ \text{J}}{1\ \text{kcal}}\right) = 14\ \text{kW}$$

ASSESS The answers seem unreasonably large. Typically, lifting weights burns around 300 kcal per hour.

61. INTERPRET This problem involves converting power from W to gallons per day.

DEVELOP From Appendix C we find that the energy content of oil is 39 kW·h/gal. Let the units guide you in converting from GW to gallons/day.

EVALUATE The import rate is

$$(800\ \text{GW})\overbrace{\left(\frac{1\ \text{gal}}{39\ \text{kW}\cdot\text{h}}\right)}^{=1}\overbrace{\left(\frac{10^6\ \text{k}}{\text{G}}\right)}^{=1}\overbrace{\left(\frac{24\ \text{h}}{\text{day}}\right)}^{=1} = 490\times10^{12}\ \text{gal/day}$$

ASSESS This may also be express as 490 Tgal/day.

71. **INTERPRET** This problem involves the concepts of power and work (or energy). Over a given period of time, the refrigerators will consume different amounts of energy, which we can calculate given their power consumption. We are to find when the cost difference for the energy consumed equals the difference in the price of the refrigerators.

DEVELOP To find the energy consumed, use Equation 6.17, $W = P\Delta t$. Thus, the work done (i.e., energy consumed) by the standard refrigerator is $W_s = P_s\Delta t_s$, where $P_s = 425$ W and $\Delta t_s = 0.20\Delta t$. The work done by the energy-efficient refrigerator in the same time interval is $W_{ee} = P_{ee}\Delta t_{ee}$, where $P_{ee} = 225$ W and $\Delta t_{ee} = 0.11\Delta t$. The cost difference Δc for the energy consumed is $\Delta c = p(W_s - W_{ee})$, where $p = 9.5$ ¢/kW·h is the price. We need to find the time interval for which the cost difference is equal to the difference in the price of the refrigerators.

EVALUATE The difference in the original price of the refrigerators is $\Delta p = \$1150 - \$850 = \$300$. The time interval to recuperate this difference is

$$\Delta p = \Delta c = p\left(P_s\Delta t_s - P_{ee}\Delta t_{ee}\right) = p\Delta t\left[(0.20)P_s - (0.11)P_{ee}\right]$$

$$\Delta t = \frac{\Delta p}{p\left[(0.20)P_s - (0.11)P_{ee}\right]} = \frac{\cancel{\$}300}{\left(\cancel{\$}0.095\ \cancel{\text{kW}}^{-1}\cdot\text{h}^{-1}\right)\left[(0.20)(0.425\ \cancel{\text{kW}}) - (0.11)(0.225\ \cancel{\text{kW}})\right]} = 5.24\times10^4\ \text{h} = 6.0\ \text{y}$$

ASSESS Notice that we converted the units so that all quantities were expressed in the same units. The answer is expressed to two significant figures because that is the least number of significant figures in the data.

81. **INTERPRET** A mass falls a given distance, and we are asked to find the force necessary to stop the mass within a another given distance. From the work-energy theorem (Equation 6.14, $\Delta K = W_{net}$), we see that the work done by gravity on the way down is equal in magnitude to the work done by the stopping force, because there is no change in kinetic energy between the initial (leg on bed) and final (leg on floor) state.

DEVELOP The height dropped is $h = 0.7$ m and the stopping distance is $s = 0.02$ m. The mass of the leg is m = 8 kg. From the work-energy theorem, we know that $|W_{down}| = |W_{stop}|$. The work done by gravity is $W_{down} = mgh$, and the absolute value of the work done by the stopping force is $|W_{stop}| = F_s s$, where F_s is the stopping force.

EVALUATE From the work-energy theorem, we have

$$\begin{aligned} |W_{down}| &= |W_{stop}| \\ mgh &= F_s s \\ F_s &= mg\frac{h}{s} \end{aligned}$$

The value $h/s = (0.7\ \text{m})/(0.02\ \text{m}) = 35$, so the average stopping force is 35 times the weight of the leg.

ASSESS The shorter the distance over which something is stopped, the greater the force required. This is why cars are built to "crumple" on impact: The increased distance traveled by the passengers during the crash means a lower average force on their bodies.

Chapter 7

10. **INTERPRET** In this problem we want to find the work done by the frictional force in moving a block from one point to another over two different paths. Friction is not a conservative force, so mechanical energy is not conserved.

DEVELOP Figure 7.15 is a plan view of the horizontal surface over which the block is moved, showing the paths (a) and (b). The force of friction is $f_k = \mu_k n = \mu_k mg$ (see Equation 5.3) and is directed opposite to the displacement. Because f_k is constant, we use Equation 6.11,

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

Because the friction force is directed opposite to the displacement $d\vec{r}$, the scalar product introduces a negative sign. For path (a), Equation 6.11 takes the form

$$W_a = -\int_{x_1}^{x_2} f_k dx - \int_{y_1}^{y_2} f_k dy = -f_k(x_2 - x_1) - f_k(y_2 - y_1)$$

where $x_1 = 0$, $y_1 = 0$, $x_2 = \text{L}$, $y_2 = \text{L}$. For path (b), we use radial coordinates, and Equation 6.11 takes the form

$$W_b = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = -f_k \Delta r$$

where $\Delta r = \sqrt{L^2 + L^2} = \sqrt{2}L$, and the scalar product gives the negative sign because friction always acts opposite to the displacement.

EVALUATE The work done by friction along path (a) is thus

$$W_a = -\mu_k mg(2L)$$

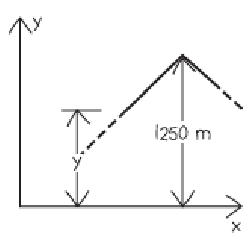
The work done by friction along path (b) is

$$W_b = -\sqrt{2}\mu_k mgL$$

ASSESS Because the work done depends on the path chosen, friction is not a conservative force.

15. **INTERPRET** The problem is about the change in gravitational potential energy as the hiker ascends. Given the position of zero potential energy, we are interested in her altitude.

DEVELOP The change in potential energy with a change in the vertical distance Δy is given by Equation 7.3, $\Delta U = mg\Delta y = mg(y - y_0)$. Knowing ΔU and y_0 allows us to determine *y*, see the figure below.



EVALUATE Equation 7.3 gives

$$\Delta U = U(y) - U(y_0) = mg(y - y_0)$$

From the above expression, we find the altitude of the hiker to be

$$y = y_0 + \frac{\Delta U}{mg} = 1250\text{ m} + \frac{-240\text{ kJ}}{(60\text{ kg})(9.8\text{ m/s}^2)} = 840\text{ m}$$

ASSESS In this problem, the point of zero potential energy is taken to be the top of the mountain with $y_0 = 1250$ m. Since the hiker's potential energy is negative, we expect the hiker's altitude to be lower than y_0.

21. **INTERPRET** This problem involves the conservative forces of gravity and the elastic force, so we can apply the conservation of mechanical energy to this problem. We are interested in finding the height to which the arrow rises, given its initial elastic potential energy.

DEVELOP We will take the initial position of the arrow to be the zero of potential energy. The initial total mechanical energy of the arrow is then just the elastic potential energy of the arrow, $U_e = kx^2/2$, with $x = 0.71$ m and $k = 430$ N/m. The final total mechanical energy of the arrow is simply the gravitational potential energy, $U_g = mg\Delta y$, because the arrow has zero speed at the peak of its trajectory, so its kinetic energy there is zero.

EVALUATE By conservation of total mechanical energy, we equate the initial and final total mechanical energies to find the height Δy to which the arrow rises. The result is

$$U_e = U_g$$

$$\frac{1}{2}kx^2 = mg\Delta y$$

$$\Delta y = \frac{kx^2}{2mg} = \frac{(430\ \text{N/m})(0.71\ \text{m})^2}{2(0.12\ \text{kg})(9.8\ \text{m/s}^2)} = 92\ \text{m/s}$$

ASSESS Notice that the height is measured from the arrow's position when the bow is taught, because that is the position at which the arrow has the elastic potential energy.

28. **INTERPRET** Water is pumped to a higher reservoir to store potential energy. We need to calculate the gravitational potential energy of the reservoir, and the time it would take to drain the reservoir given the power output of the generators. Although it is not stated in the problem, we will assume that the efficiency of the generators is 100%.

DEVELOP The mass of the reservoir is $m = 2.1\times10^{10}$ kg, and the height above the generators is $h = 214$ m. The initial gravitational potential energy is $U_0 = mgh$, since the level of the generators is taken to be $U = 0$. As the reservoir drains, there will be less water and therefore less potential energy. This loss in potential energy goes into kinetic energy of the water, which does work on the generators: $W = \Delta K = -\Delta U$. The maximum work possible corresponds to draining the whole reservoir of water (letting it all flow down to $U = 0$), which is equivalent to using up all the initial potential energy:

$$W_{\text{max}} = -\Delta U_{\text{total}} = -(0 - U_0) = U_0$$

If the power is constant, then the amount of work done is $W = P\Delta t$ (recall Equation 6.17), so the time to drain the whole reservoir is $\Delta t = U_0 / P$.

EVALUATE (a) The total potential energy of the reservoir is

$$U_0 = mgh = (2.1\times10^{10}\ \text{kg})(9.8\ \text{m/s}^2)(214\ \text{m}) = 4.4\times10^{13}\ \text{J}$$

(b) The total time the generators can run before the reservoir is empty can be found from the equation above:

$$\Delta t = \frac{U_0}{P} = \frac{4.4\times10^{13}\ \text{J.}}{1.08\ \text{GW}} = 11\ \text{h}$$

ASSESS The energy stored in the full reservoir is equivalent to about 12 million kWh of electricity, or 12 GWh. As explained in the text, reservoirs such as this are often used to store power that can be used during periods of peak demand. One can imagine this facility generating power during the daylight hours, and then "recharging" (pumping water back up to the top reservoir) during the night.

34. **INTERPRET** We're asked to characterize the Achilles tendon as a mechanical spring.

DEVELOP When a 125-kg mass is hung on the tendon, it stretched until its restorative spring force countered the mass' weight: $kx = mg$. From this, we can find the spring constant. And furthermore, we can use Equation 7.4, $U = \frac{1}{2}kx^2$, to find the distance the tendon must stretch in order to store 50.0 J of energy.

EVALUATE (a) The experiment with the mass is enough to tell us what the spring constant is for the tendon:

$$k = \frac{mg}{x} = \frac{(125\text{ kg})(9.8\text{ m/s}^2)}{2.66\text{ mm}} = 4.605\times10^5\text{ N/m} \simeq 461\text{ N/mm}$$

(b) In order to store 50.0 J in the tendon, it must be stretched by

$$x = \sqrt{\frac{2U}{k}} = \sqrt{\frac{2(50.0\text{ J})}{(4.605\times10^5\text{ N/m})}} = 14.7\text{ mm}$$

Note that we put k in units of N/m in order to avoid confusion in the units, since $\text{J/(N/m)} = \text{m}^2$.

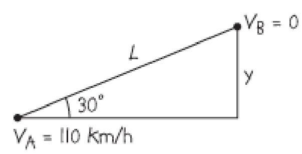
ASSESS In general, tendons connect muscle to bone. The Achilles tendon, in particular, connects the muscles in the lower leg to the heel bone. It therefore has to withstand forces equal to and greater than that of a person's body weight without stretching too much. The relatively large spring constant that we found in part (a) bears witness to that fact.

38. **INTERPRET** The problem is about conservation of mechanical energy. Both potential energy and kinetic energy are involved. The kinetic energy of the truck is converted to gravitational potential energy as it moves uphill.

DEVELOP Initially the kinetic energy of the truck is $K_0 = \frac{1}{2}mv_A^2$, and you can set its gravitational potential energy as $U_0 = 0$. In the final state, all the kinetic energy has been converted to gravitational potential energy: $K = 0$ and $U = mgy$. These quantities are related by the principle of conservation of mechanical energy given in Equation 7.7: $K_0 + U_0 = K + U$, which gives you:

$$\frac{1}{2}mv_A^2 + 0 = 0 + mgy$$

You are asked to find how long the lane should be in order to stop a runaway truck going $v_A = 110\text{ km/h} = 30.6\text{ m/s}$. The length of the lane, L, is related to the height by: $L = y/\sin\theta$, see the figure below.



EVALUATE Solving for the length, you have:

$$L = \frac{v_A^2}{2g\sin\theta} = \frac{(30.6\text{ m/s})^2}{2(9.8\text{ m/s}^2)\sin 30°} = 95\text{ m}$$

ASSESS Notice that the mass of the truck is irrelevant. The distance the runaway truck travels only depends on its velocity and the angle of the incline. If you make the incline steeper, the truck will still climb to the same height, y, but the length of the lane can be made shorter.

52. **INTERPRET** You're asked to determine the efficiency of a pumped storage facility. If the efficiency were 100%, then all the gravitational potential energy of the water would be converted to electricity. So you have to find what actual percentage of the potential energy is converted to electricity.

DEVELOP Initially, all the water is in the upper reservoir. If we assume the gravitational potential energy is zero at the level of the generating station, then the stored potential energy of the water in the upper reservoir is $U = mgh$. You need to compare this to the electrical energy generated by the station over the given time period: $W = P\Delta t$.

EVALUATE The efficiency is the energy output of the generators divided by the energy input of the water:

$$\varepsilon = \frac{W}{U} = \frac{P\Delta t}{mgh} = \frac{(330\text{ MW})(8.0\text{ h})}{(8.5\times10^9\text{ kg})(9.8\text{ m/s}^2)(140\text{ m})} = 81\%$$

ASSESS The missing energy is lost to non-conservative forces, such as drag forces in pipes that channel the water and friction in the turbines that turn the generators.

62. **INTERPRET** This problem involves only conservative forces (i.e., gravity), so we can apply conservation of total mechanical energy. We will take the zero of the gravitational potential energy to be the ground level. We are asked to find the speed at which Tarzan must run so that he swings just far enough to cross the gorge, so that he can just let go and drop vertically down on the far side.

DEVELOP Tarzan's initial mechanical energy is completely kinetic, so $U_0 + K_0 = mv_0^2/2$. His final mechanical energy will be completely due to gravitational potential energy, so $U + K = mgy$. Using trigonometry, the final height h can be expressed in terms of the width $w = 10$ m of the gorge and the length $l = 17$ m of the vine: $h = l(1 - \cos\theta)$, where $\text{asin}(d/l) = \theta$.

EVALUATE Equating the initial and final energies, and solving for the speed v gives

$$\frac{1}{2}mv_0^2 = mgy = mgl(1-\cos\theta) = mgl\left(1-\frac{\sqrt{l^2-d^2}}{l}\right)$$

$$v_0 = \pm\sqrt{2gl\left(1-\frac{\sqrt{l^2-d^2}}{l}\right)} = \pm\sqrt{2\left(9.8\text{ m/s}^2\right)\left(17\text{ m}\right)\left(1-\frac{\sqrt{(17\text{ m})^2-(10\text{ m})^2}}{17\text{ m}}\right)} = 8.0\text{ m/s}$$

to two significant figures and where we have chosen the positive sign to indicate that Tarzan must run to the right.

ASSESS If we let the gorge shrink to $d = 0$, we find that v_0 goes to zero, as expected. The two signs indicate that the speed can be to the right or to the left, but we have chosen the positive direction to be to the right.

64. **INTERPRET** How long will it take to accelerate from zero to 60 mph? We are given the power of the car, the efficiency, and the mass. When the car travels on a level road, the work done equals the change in kinetic energy (Equation 7.5). Power is work per time, so we can find the time from the power and the work.

DEVELOP Convert the given power of the car (250 hp) to Watts, using 1 hp = 746 W, recalling that the power available is only 30% of the engine horsepower. Also convert the final speed from mph to m/s, using 1 mph = 0.447 m/s. Use $P = W/t = \Delta K/t$, and solve for the time t.

EVALUATE The power of the car is $P = (250\text{ hp})\times(30\%)\times(746\text{ W}/1\text{ hp}) = 56\text{ kW}$. The final speed of the car is $v = (60\text{ mph})\times(0.447\text{ m/s}/1\text{ mph}) = 26.8\text{ m/s}$. The time to reach this speed is

$$P = \frac{W}{t} = \frac{\Delta K}{t}$$

$$t = \frac{\Delta K}{P} = \frac{\frac{1}{2}mv^2}{P} = \frac{(1500\text{ kg})(26.8\text{ m/s}))^2}{2\left(56\times10^3\text{ W}\right)} = 9.6\text{ s}$$

ASSESS This is not particularly high performance, but it's a reasonable 0-to-60 time for an automobile.

66. **INTERPRET** This problem involves conservative forces (i.e., gravity), so we can apply conservation of total mechanical energy. We are to derive a "leaping equation" that relates the power of the animal to its mass, the push-off distance, and the height reached. We will take the ground to be the zero of gravitational potential energy.

DEVELOP The height when the animal leaves the ground is d, and the final height attained in the jump is h. To apply the conservation of total mechanical energy, we equate the mechanical energy at the heights d and h. At d, the energy is $U_0 + K_0 = mgd + mv^2/2$, and at h the energy is $U + K = mgh + 0$. From Equation 6.12 we know that the change in kinetic energy equates to the *net* work done, which is the work done by the animal plus the work done by gravity. Thus, we know that to accelerate from zero to v, the animal must do work given by

$$W_{\text{net}} = W_{\text{anmial}} + \overbrace{W_{\text{gravity}}}^{=-mgd} = \Delta K = \frac{1}{2}mv^2 - 0$$

$$W_{\text{animal}} = \frac{1}{2}mv^2 + mgd$$

where the work done by gravity is negative because the force of gravity acts to oppose the animal's upward movement. Power is work per unit time ($P = W/t$), and we use Equation 2.9:, $d = (v_0 + v)t/2$ to find t, with $v_0 = 0$, so $t = 2d/v$.

EVALUATE Conservation of mechanical energy gives the kinetic energy at d:

$$mgd+\frac{1}{2}mv^2=mgh \quad\Rightarrow\quad v=\pm\sqrt{2g(h-d)}$$

Power is

$$P=\frac{W_{\text{animal}}}{t}=\frac{mv^2/2+mgd}{2d/v}=\frac{mgh}{2d}\sqrt{2g(h-d)}$$

ASSESS We see that the power depends linearly on the mass, and in a more complex manner on the h and d.
We'd better check units on this equation:

$$P=\frac{\text{kg}\cdot\text{m}\cdot\text{s}^{-2}\cdot\text{m}}{\cancel{\text{m}}}\sqrt{\cancel{\text{m}}\cdot\text{s}^{-2}\cdot\cancel{\text{m}}}=\overbrace{\underbrace{\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right)}_{=\text{N}}\cdot\text{m}}^{=\text{J}}\cdot\text{s}^{-1}=\text{J/s}$$

It's good!

Chapter 8

17. **INTERPRET** We're asked to find the height of the building by using the difference in the gravitational acceleration at the top and bottom of the building. The change in the acceleration is due to the change in the distance to the center of the Earth.

DEVELOP In general, the acceleration due to gravity is given in Equation 8.2: $a=GM/r^2$. The acceleration is measured at street level, where $r=R_E$, and compared to reading at the top of the Willis Tower, where $r=R_E+h$. Here, R_E is the radius of the Earth, and h is the height of the building. The difference in the acceleration measurements should equal:

$$\Delta a=\frac{GM_E}{R_E^2}-\frac{GM_E}{(R_E+h)^2}=\frac{GM_E}{R_E^2}\left[1-\frac{1}{(1+h/R_E)^2}\right]$$

Since $h\ll R_E$, we can use the binomial approximation from Appendix A: $(1+h/R_E)^{-2}\approx 1-2h/R_E$. The above expression reduces to: $\Delta a\approx 2gh/R_E$, where we have used $g=GM_E/R_E^2$ for the average value of the gravitational acceleration on the Earth's surface.

EVALUATE Using the above expression for the acceleration difference, we solve for the height of the tower:

$$h\approx R_E\frac{\Delta a}{2g}=(6.37\times10^6\,\text{m})\frac{(1.36\text{ mm/s}^2)}{2(9.8\text{ m/s}^2)}=442\text{ m}$$

ASSESS The 108-story Willis Tower is indeed 442 m tall. Note that present gravimeters can measure differences in the gravitational acceleration as small as a few tenths of a milligal, where $1\text{ milligal}=10^{-5}\,\text{m/s}^2$ is the unit used to measure gravity anomalies by geologists.

23. **INTERPRET** We're asked to find the altitude of a spacecraft orbiting Mars given its orbital period.

DEVELOP Equation 8.4 relates the period and radius of an orbiting body to the mass of the object it is orbiting around: $T^2=4\pi^2r^3/GM$. The mass of Mars from Appendix E is: $M_M=6.42\times10^{23}\,\text{kg}$. Once we solve for the orbital radius, we will have to subtract the radius of Mars $(R_M=3.38\times10^6\,\text{m})$ to find the altitude: $h=r-R_M$.

EVALUATE The distance between the Mars Renaissance Orbiter and the center of the planet Mars is

$$r=\sqrt[3]{\tfrac{1}{4\pi^2}GMT^2}=\sqrt[3]{\tfrac{1}{4\pi^2}\left(6.67\times10^{-11}\,\tfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(6.42\times10^{23}\,\text{kg})(112\cdot60\text{ s})^2}=3.659\times10^6\,\text{m}$$

This implies that the altitude of the spacecraft is

$$h=r-R_M=3.659\times10^6\,\text{m}-3.38\times10^6\,\text{m}=0.28\times10^6\,\text{m}$$

ASSESS This is 280 km. Compare this to Example 8.2, where it was shown that a low Earth orbit with an altitude of 380 km has a period of about 90 min. Since Mars has less mass, spacecrafts must orbit at a smaller radius in order to have roughly the same orbital period.

25. **INTERPRET** This problem deals with the gravitational potential energy of an object. We are asked to find the energy required to raise an object to a given height in the Earth's gravitational field.

DEVELOP If we neglect any kinetic energy differences associated with the orbital or rotational motion of the Earth or package, the required energy is just the difference in gravitational potential energy given by Equation 8.5, $\Delta U = GM_E m\left[R_E^{-1} - (R_E + h)^{-1}\right]$, where $h = 1800$ km $= 1.8 \times 10^6$ m.

EVALUATE Evaluating the expression above with the data from Appendix E gives

$$\Delta U = \left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(5.97\times10^{24}\ \mathrm{kg}\right)(230\ \mathrm{kg})\left[\frac{1}{6.37\times10^6\ \mathrm{m}} - \frac{1}{6.37\times10^6\ \mathrm{m}+1.8\times10^6\ \mathrm{m}}\right] = 3.17\ \mathrm{GJ}$$

ASSESS In terms of the more convenient combination of constants $GM_E = gR_E^2$, $\Delta U = mgR_E h/(R_E + h) = 3.17$ GJ

35. **INTERPRET** This problem explores the gravitational acceleration of a gravitating body as a function of altitude h.

DEVELOP Using Equation 8.1, the gravitational force between a mass m and a planet of mass M_p is $F = GM^p m/r^2$ where r is their separation, measured from the center of the planet. From Newton's second law (for constant mass), $F = ma$, the acceleration of gravity at any altitude $h = r - R_p$ above the surface of a spherical planet of radius R_p, is

$$g(h) = \frac{GM_p}{(R_p + h)^2} = \frac{GM_p}{R_p^2}\left(\frac{R_p}{R_p + h}\right)^2 = g(0)\left(\frac{R_p}{R_p + h}\right)^2$$

where $g(0)$ is the value at the surface. Once the ratio $g(h)/g(0)$ is known, we can find the altitude h in terms of R_p.

EVALUATE Solving for h, we find

$$\frac{h}{R_p} = \sqrt{\frac{g(0)}{g(h)}} - 1$$

Therefore, for $g(h)/g(0) = 1/2$, we have $h/R_p = \sqrt{2} - 1 = 0.414$.

ASSESS To see if the result makes sense, we take the limit $h = 0$, where the object rests on the surface of the planet. In this limit, we recover $g(0)$ as the gravitational acceleration. The equation also shows that $g(h)$ decreases as the altitude h is increased, and $g(h)$ approaches zero as $h \to \infty$.

40. **INTERPRET** In this problem we are asked to find the speed and period of an object orbiting about a gravitating body—a white dwarf.

DEVELOP Newton's law of universal gravitation describes the force between the spaceship and the white dwarf that provides the centripetal force for the spaceship to move in a circular path about the white dwarf:

$$F = \frac{GMm}{r^2} = ma_c = \frac{mv^2}{r}$$

where we used Equation 5.1, $a_c = v^2/r$, for centripetal acceleration. Solving for the orbital speed gives $v = \sqrt{GMm/r}$ (Equation 8.3). The period may be found by dividing the orbital circumference by the orbital velocity. This gives $T^2 = 4\pi^2 r^3/(GM)$ (Equation 8.4). Use the data from Appendix E to evaluate these formulas.

EVALUATE (a) The radius of a low orbit is approximately the radius of the white dwarf, or R_E, so Equation 8.3 gives

$$v = \sqrt{\frac{GM}{R_E}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(1.99\times10^{30}\ \mathrm{kg}\right)}{6.37\times10^{6}\ \mathrm{m}}} = 4.56\times10^{6}\ \mathrm{m/s}$$

or about 1.5% of the speed of light.

(b) The orbital period is

$$T = \frac{2\pi R_E}{v} = \frac{2\pi\left(6.37\times10^{6}\ \mathrm{m}\right)}{4.56\times10^{6}\ \mathrm{m/s}} = 8.77\ \mathrm{s}$$

which is very short.

ASSESS According to Kepler's third law, the relationship between T and M is given by

$$T^2 = \frac{4\pi^2 r^3}{GM} \quad \Rightarrow \quad MT^2 = \frac{4\pi^2 R^3}{G} = \text{constant}$$

Thus, we see that if the mass of the gravitating body M is increased while keeping its radius R constant, then its period T must decrease.

46. **INTERPRET** This problem involves conservation of total mechanical energy. We are to find the speed of an object as it hits the Sun, given that it starts from rest 1 AU (astronomical unit, see previous problem) from the Sun.

DEVELOP The canisters start at rest 1 AU from the Sun and free from the Earth's gravitational field. The change of potential energy as the waste canister travels into the Sun can be calculated by using conservation of total mechanical energy, $\Delta U + \Delta K = 0$ (Equation 7.6). The change in potential energy may be obtained from Equation 8.6:

$$\Delta U = U_{\text{final}} - U_{\text{initial}} = -\frac{GMm}{R_S} + \frac{GMm}{r_{ES}} = -GMm\left(\frac{1}{R_S} - \frac{1}{r_{ES}}\right)$$

where R_S is the radius of the Sun and r_{ES} is the radius of the Earth's orbit about the Sun. The gain in kinetic energy is $\Delta K = K_{\text{final}} - K_{\text{initial}} = mv^2/2 - 0 = mv^2/2$.

EVALUATE Solving Equation 7.6 for the speed v, we obtain

$$\begin{aligned} v &= \sqrt{2GM\left(\frac{1}{R_S} - \frac{1}{r_{ES}}\right)} \\ &= \sqrt{2\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(1.99\times10^{30}\ \mathrm{kg}\right)\left(\frac{1}{6.96\times10^{8}\ \mathrm{m}} - \frac{1}{1.50\times10^{11}\ \mathrm{m}}\right)} \\ &= 616\ \mathrm{km/s} \end{aligned}$$

ASSESS How much energy per kg would be required to implement this solution? This may be found by using (again) conservation of total mechanical energy. The energy per kg to escape the Earth's gravitational field is (using Equation 8.6)

51. **Interpret** The question boils down to: is the comet's orbit open or closed? Will it orbit around the Sun multiple times (and therefore pass by the Earth again), or is it destined to escape our solar system?

Develop The comet's orbit is open if the given velocity is greater than the escape velocity: $v = \sqrt{2GM/r}$. In this case, the mass is the Sun and the radius is the Earth's distance from the Sun.

Evaluate The escape velocity from the Sun at Earth's orbital radius is

$$v_{\text{esc}} = \sqrt{\frac{2GM_S}{r_E}} = \sqrt{\frac{2\left(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(1.99\times10^{30}\,\text{kg}\right)}{\left(150\times10^{9}\,\text{m}\right)}} = 42.1\ \text{km/s}$$

The comet is going faster than this escape velocity, so it is on a open (hyperbola) orbit, see Figure 8.9. It will not return to Earth's vicinity.

Assess The escape velocity calculated here is much smaller than the 618 km/s escape velocity given in Appendix E for the Sun. But the larger value is the escape velocity from the Sun's surface. Farther away at the Earth's orbital radius the escape velocity doesn't need to be so high. Also note that the comet velocity is much greater than the Earth's escape velocity of 11.2 km/s, which just means that there's no danger of the comet getting captured in a closed orbit around Earth.

65. **Interpret** This problem involves Kepler's third law, which we will apply to convert the rate of change in the orbital period of the Moon to the rate of change in its orbital distance (i.e., its radial speed). We assume that the Moon's orbit is approximately circular for this calculation.

Develop Kepler's third law relates the orbital period to the semimajor axis of an elliptical orbit (of which a circular orbit is a special case): $T^2 = 4\pi^2 r^3/(GM)$. We are told the rate of change is

$$\frac{dT}{dt} = \left(35\times10^{-3}\,\frac{\text{s}}{100\ \text{y}}\right)\left(\frac{1\ \text{y}}{365\ \text{d}}\right)\left(\frac{1\ \text{d}}{86,400\ \text{s}}\right) = 1.11\times10^{-11}$$

so we can differentiate Kepler's law to find the rate of change in the orbital radius r.

Evaluate Differentiating Kepler's law gives

$$\frac{dT}{dt} = \pm\frac{d}{dT}\sqrt{\frac{4\pi^2 r^3}{GM}} = \frac{3}{2}\sqrt{\frac{4\pi^2 r}{GM}}\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{dT}{dt}\left(\frac{2}{3}\sqrt{\frac{GM}{4\pi^2 r}}\right) = \left(1.11\times10^{-11}\right)\left(\frac{2}{3}\sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\ \text{kg}\right)}{4\pi^2\left(3.85\times10^{8}\ \text{m}\right)}}\right)$$

$$= \left(1.33\times10^{-10}\ \text{m/s}\right)\left(\frac{86,400\ \text{s}}{1\ \text{d}}\right)\left(\frac{36500\ \text{d}}{1\ \text{c}}\right)\left(\frac{100\ \text{cm}}{1\ \text{m}}\right) = 41.9\ \text{cm/c}$$

where we have taken the positive square root because the orbital radius is increasing, not decreasing.

Assess At this speed, we don't have to worry about the Moon leaving any time soon!