Chapter 9

12. **INTERPRET** This one-dimensional problem involves finding the center of mass of a system with two objects (child and father).

DEVELOP In one dimension, Equation 9.2 for the center of mass reduces to

$$x_{cm} = \frac{\sum_i m_i x_i}{M} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

Taking the origin of the coordinate system to be at the child (who we denote with subscript 1), we have $x_1 = 0$ and $m_1 = 28$ kg. The center of the seesaw is then at $x_{cm} = 3.5/2$ m = 1.75 m (where we retain an extra significant figure because this is an intermediate result). The position of the father is the unknown, and is labeled x_2. The mass of the father is $m_2 = 65$ kg.

EVALUATE Inserting the known quantities into the expression for center of mass gives

$$x_{cm} = \frac{\overbrace{x_1}^{=0} m_1 + x_2 m_2}{m_1 + m_2}$$

$$x_2 = \frac{x_{cm}(m_1 + m_2)}{m_2} = \frac{(1.75\text{ m})(28\text{ kg} + 65\text{ kg})}{65\text{ kg}} = 2.5\text{ m}$$

from the child.

ASSESS The algebra was simplified somewhat by choosing the origin of the coordinate system to be at under the child's posterior. Because $x_2 < 3.5$ m, the father can sit on the seesaw and balance it with his daughter. If $m_2 \gg m_1$, then $x_2 = x_{cm}$, because it does not really matter where the child sits if the father weighs a ton!

20. **INTERPRET** This problem involves using conservation of linear momentum to find the final speed of a moving toboggan after some snow drops onto it.

DEVELOP Because there is no net external horizontal force, the total momentum of the snow-toboggan system is conserved. The initial momentum of the system is $P_i = m_t v_{ti}$. Because the snow and the toboggan move together with the same speed v_f, the final momentum is $P_f = (m_t + m_s) v_f$.

EVALUATE By conservation of momentum, $P_i = P_f$, the final speed of the toboggan-snow system is

$$v_f = \frac{m_t}{m_t + m_s} v_{ti} = \frac{8.6\text{ kg}}{8.6\text{ kg} + 15\text{ kg}}(23\text{ km/h}) = 8.4\text{ km/h}$$

ASSESS To see that our result makes sense, let's consider the following limiting cases: (i) $m_s = 0$. In this situation, we have $v_f = v_{ti}$, which indicates that the toboggan continues with the same speed. (ii) $m_s \to \infty$. In the situation where a large quantity of snow is dumped onto the toboggan, we expect the system to slow down considerably, which is indeed is what our equation gives ($v_f = 0$).

21. **INTERPRET** In this problem we are asked about the energy gained by the baseball pieces after the baseball explodes. We can apply conservation of linear momentum to solve this problem.

DEVELOP Applying conservation of linear momentum to the baseball gives

$$\vec{P}_i = \vec{P}_f \quad \Rightarrow \quad (m_1 + m_2)\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2$$

The initial kinetic energy of the system is $K_i = \frac{1}{2}(m_1 + m_2)v_0^2$, and the total final kinetic energy is $K_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$. Therefore, the change in kinetic energy is

$$\Delta K = K_f - K_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}(m_1 + m_2)v_0^2 = \frac{1}{2}m_1(v_1^2 - v_0^2) + \frac{1}{2}m_2(v_2^2 - v_0^2)$$

EVALUATE Let the forward direction be positive. By conservation of momentum, the velocity of the second piece, with mass $m_2 = m - m_1 = 150\text{ g} - 38\text{ g} = 112\text{ g}$, is

$$v_2 = \frac{(m_1 + m_2)v_0 - m_1v_1}{m_2} = \frac{(150\text{ g})(60\text{ km/h}) - (38\text{ g})(85\text{ km/h})}{112\text{ g}} = 51.5\text{ km/h} = 14.3\text{ m/s}$$

In SI units $v_0 = 16.67\text{ m/s}$ and $v_1 = 23.6\text{ m/s}$, so the difference in kinetic energy is

$$\begin{aligned}\Delta K = \Delta K_1 + \Delta K_2 &= \frac{1}{2}m_1(v_1^2 - v_0^2) + \frac{1}{2}m_2(v_2^2 - v_0^2) \\ &= \frac{1}{2}(38\times10^{-3}\text{ kg})\left[(23.6\text{ m/s})^2 - (16.7\text{ m/s})^2\right] + \frac{1}{2}(112\times10^{-3}\text{ kg})\left[(14.3\text{ m/s})^2 - (16.7\text{ m/s})0^2\right] \\ &= 1.21\text{ J}\end{aligned}$$

ASSESS The change in kinetic energy for the first piece (ΔK_1) is positive because $v_1 > v_0$, but negative for the second ($\Delta K_2 < 0$ because $v_2 < v_0$).

24. **INTERPRET** We want to determine the average force and impulse acting on a jumping flea.

DEVELOP We're given the average acceleration during the jump, so the ground must supply an average force on the flea of $\bar{F} = m\bar{a}$ is just multiplied by the flea's mass. We can then use Equation 9.9a $(J = \bar{F}\Delta t = \Delta p)$ to find the impulse imparted by the ground and the resulting momentum change for the flea.

EVALUATE (a) The average force exerted by the ground on the flea is

$$\bar{F} = m\bar{a} = (220\times10^{-9}\text{kg})(100\cdot 9.8\text{ m/s}^2) = 2.16\times10^{-4}\text{N} \simeq 220\ \mu\text{N}$$

(b) Multiplying the average force by the time gives the impulse:

$$J = \bar{F}\Delta t = (2.16\times10^{-4}\text{N})(1.2\text{ ms}) = 2.6\times10^{-7}\text{N}\cdot\text{s}$$

(c) The momentum change for the flea is equal to the impulse provided by the floor:

$$\Delta p = J = 2.6\times10^{-7}\text{N}\cdot\text{s} = 2.6\times10^{-7}\text{kg}\cdot\text{m/s}$$

Notice that we can write the momentum change in units $(\text{kg}\cdot\text{m/s})$ that might be more familiar for momentum.

ASSESS If we assume the flea starts its jump from rest, then at the end of its jump it reaches a velocity of $v = \Delta v = \Delta p / m = 1.2\text{ m/s}$. That seems reasonable.

29. **INTERPRET** This is a totally inelastic collision, since the trucks move together as one after they hit. You should be able to find the mass of the second truck using conservation of momentum.

DEVELOP According to Equation 9.11, conservation of momentum in the truck collision implies

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}_f$$

EVALUATE The first truck is at rest $(\vec{v}_1 = 0)$, which means the final velocity has to be in the same direction as $\vec{v}_2$. Given that the first truck has a mass of $m_1 = 5500\text{ kg} + 3800\text{ kg} = 9300\text{ kg}$, we can solve for the mass of the second:

$$m_2 = m_1\frac{v_f}{v_2 - v_f} = (9300\text{ kg})\frac{40\text{ km/h}}{65\text{ km/h} - 40\text{ km/h}} = 14{,}880\text{ kg}$$

Subtracting the mass of the truck leaves a load of 9400 kg, so the second truck was overloaded by 1400 kg.

ASSESS If the truck had been loaded at the permissible limit of 8000 kg, the final velocity after the collision would have been 38 km/h.

31. **INTERPRET** This problem is about head-on (i.e. one-dimensional) elastic collisions. We want to find the speed of the ball after it rebounds elastically from a moving car.

DEVELOP Both mechanical energy and linear momentum are conserved in an elastic collision. In this one-dimensional case, conservation of linear momentum gives

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Conservation of energy gives

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Using the two conservation equations, the final speeds of m_1 and m_2 are (see Equations 9.15a and 9.15b):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

EVALUATE Let the subscripts 1 and 2 be for the car and the ball, respectively. We choose positive velocities in the direction of the car. The speed of the ball after it rebounds is

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} \approx 2v_{1i} - v_{2i} = 2(14\ \text{m/s}) - (-18\ \text{m/s}) = 46\ \text{m/s}$$

where we have used $m_1 \gg m_2$.

ASSESS Similarly, the final speed of the car is

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \approx v_{1i} = 14\ \text{m/s}$$

36. **INTERPRET** This problem concerns stopping a charging rhino with rubber bullets that lose all their momentum when they hit the animal.

DEVELOP The impulse imparted on the rhino by one bullet is equal to the rhino's change in momentum (Equation 9.10a: $\vec{J}_b = \Delta\vec{p}_r$). But we don't know the mass of the rhino, so it is easier to deal with the bullets. Their change in momentum is equal and opposite to the change in momentum of the rhino: $\Delta p_r = -\Delta p_b$. Note that we've dropped the vector notation, since the momenta are collinear, but we'll assume that the bullets are initially moving in the positive direction. We're given the initial velocity of the bullets, and we know that they fall straight to the ground after impact, so their final velocity must be zero. Putting all this together, we have:

$$J_b = \Delta p_r = -\Delta p_b = -(0 - m_b v_{b0}) = m_b v_{b0}$$

We can find the mass of the rhino by calculating the total impulse supplied by all the bullets fired at the rhino: $J_{tot} = N_b J_b$. This total impulse is what supposedly brings the rhino to rest: $J_{tot} = \Delta p_{r,tot} = 0 - m_r v_{r0} = -m_r v_{r0}$. The minus sign is not a problem, since the rhino's initial velocity is negative compared to the positive velocity of the bullets.

EVALUATE (a) From the expression above, the impulse imparted by one bullet is

$$J_b = m_b v_{b0} = (20\ \text{g})(73\ \text{m/s}) = 1.46\ \text{kg}\cdot\text{m/s} \simeq 1.5\ \text{N}\cdot\text{s}$$

(b) To find the mass of the rhino, we first need to calculate the number of bullets, which is the rate the gun is fired multiplied by the time:

$$N_b = rt = (15\ \text{bullets/s})(34\ \text{s}) = 510\ \text{bullets}$$

The mass can then be found from the total impulse from all these bullets:

$$m_r = \frac{\Delta p_{r,tot}}{-v_{r0}} = \frac{N_b J_b}{-v_{r0}} = \frac{(510)(1.46\ \text{kg}\cdot\text{m/s})}{-(-0.81\ \text{m/s})} = 920\ \text{kg}$$

ASSESS The answer is reasonable for a black rhino. But note that white rhinos have typically twice this mass.

54. **INTERPRET** This problem involves exerting a force on a conveyor belt to compensate for the change in momentum caused by the drops of cookie dough that drop onto the belt.

DEVELOP If the conveyor belt is horizontal and moving with speed $v = 50$ cm/s and the mounds of dough fall vertically, then the change in the horizontal momentum due to each mound of mass Δm is $\Delta p = (\Delta m)v$. The average horizontal force needed is equal to the rate at which mounds are dropped (a number N in time Δt, or $N/\Delta t$) times the change in momentum due to a single mound. Thus, for this problem Equation 9.6 takes the form

$$\vec{F}_{\rm av} = \left(\frac{N}{\Delta t}\right)\Delta\vec{p} = \left(\frac{N}{\Delta t}\right)(\Delta m)\vec{v}$$

EVALUATE Inserting the values given in the problem statement, we find that the average force the conveyor belt exerts on a cookie sheet is

$$F_{\rm av} = \left(\frac{N}{\Delta t}\right)(\Delta m)v = \left(\frac{1}{2\ \text{s}}\right)(0.012\ \text{kg})(0.50\ \text{m/s}) = 3.0\times10^{-3}\ \text{N}$$

ASSESS The average force is just the total change in momentum, $\Delta\vec{P} = N\Delta\vec{p} = N(\Delta m)\vec{v}$, divided by the time, Δt. The greater is the change in momentum over a given time interval, the greater is the average force.

63. **INTERPRET** This two-dimensional problem asks for the speed of one of two vehicles just before its totally inelastic collision with the second vehicle. Given the road condition (i.e., the coefficient of kinetic friction), we want to show that the speed of one of the cars exceeded 25 km/h. Energy is not conserved in this process, but momentum is. Furthermore, because work is done by friction, this problem involves the work-energy theorem.

DEVELOP If the wreckage skidded on a horizontal road, the work-energy theorem requires that the work done by friction be equal to the change of the kinetic energy of both cars. $W_{\rm nc} = \Delta K$ (see Equation 7.5). Because $W_{\rm nc} = -f_{\rm k}x = -\mu_{\rm k}nx = -\mu_{\rm k}(m_1+m_2)gx$, and $\Delta K = K_{\rm f} - K_{\rm i} = 0 - \frac{1}{2}(m_1+m_2)v^2$, where v is the speed of the wreckage immediately after collision, we are led to

$$\mu_{\rm k}gx = \frac{1}{2}v^2$$

Therefore, the speed of the wreckage just after the collision is $v = \pm\sqrt{2\mu_{\rm k}gx}$. Next, momentum conservation requires that the initial and final momentum are the same, so

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{v}$$
$$\vec{v} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$$

where $\vec{v}$ is the initial velocity of the wreckage. To find the change in kinetic energy, we need to calculate the scalar product $\vec{v}\cdot\vec{v}$:

$$v^2 = \vec{v}\cdot\vec{v} = \left(\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}\right)\cdot\left(\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}\right)$$
$$= \frac{m_1^2v_1^2 + m_2^2v_2^2}{(m_1+m_2)^2} + \frac{2m_1m_2\overbrace{\vec{v}_1\cdot\vec{v}_2}^{=0}}{(m_1+m_2)^2}$$
$$= \frac{m_1^2v_1^2 + m_2^2v_2^2}{(m_1+m_2)^2}$$

where we have used the fact that the scalar product $\vec{v}_1\cdot\vec{v}_2 = 0$ because the initial velocities are perpendicular to each other. In the next step, we insert the maximum speed for one car to find the minimum speed for the other car.

EVALUATE Inserting $v = \pm\sqrt{2\mu_{\rm k}gx}$ into the above expression for the initial velocity of the wreckage leads to

$$v^2 = \frac{m_1^2v_1^2 + m_2^2v_2^2}{(m_1+m_2)^2} = 2\mu_{\rm k}gx$$

Solving for v_1 gives

$$v_1 = \sqrt{\frac{2\mu_{\rm k}gx(m_1+m_2)^2 - m_2^2v_2^2}{m_1^2}}$$

where we have taken the positive square root. Consider now the following situations:

Let subscript 1 correspond to the Toyota 2 to the Buick. If the speed of the Buick is $v_2 = 25$ km/h = 6.94 m/s, then the speed of the Toyota would be

$$v_1 = \sqrt{\frac{2\mu_k gx(m_1+m_2)^2 - m_2^2 v_2^2}{m_1^2}}$$
$$= \sqrt{\frac{2(0.91)(9.8\ \text{m/s}^2)(22\ \text{m})(1200\ \text{kg}+2200\ \text{kg})^2 - (2200\ \text{kg})^2(6.94\ \text{m/s})^2}{(1200\ \text{kg})^2}}$$
$$= 55\ \text{m/s} = 200\ \text{km/h}$$

Thus, we conclude that the speed of the Toyota exceeded 25 km/h.

(2) Here, we reverse the assignment of the subscripts 1 and 2. If the speed of the Toyota is $v_2 = 25$ km/h = 6.94 m/s, then the speed of the Buick would be

$$v_1 = \sqrt{\frac{2\mu_k gx(m_1+m_2)^2 - m_2^2 v_2^2}{m_1^2}}$$
$$= \sqrt{\frac{2(0.91)(9.8\ \text{m/s}^2)(22\ \text{m})(2200\ \text{kg}+1200\ \text{kg})^2 - (1200\ \text{kg})^2(6.94\ \text{m/s})^2}{(2200\ \text{kg})^2}}$$
$$= 30\ \text{m/s} = 110\ \text{km/h}$$

Thus, we conclude that the speed of the Buick exceeded 25 km/h.

From the analysis above, we conclude that if one car is going at 25 km/h, then the other one must have been speeding, so at least one car must have been speeding.

ASSESS If we knew the direction of the wreckage velocity, we could easily find the car that was speeding.

76. INTERPRET This one-dimensional two-body problem involves an elastic collision and kinematics. We can apply conservation of energy and momentum to find the height the small ball rebounds after being dropped together with a larger ball and rebounding from the ground.

DEVELOP The balls reach the ground, after a vertical fall through a height *h*, with speed $v_0 = \sqrt{2gh}$ (see Equation 2.11). Assume that they undergo an elastic head-on collision, with the large ball *M* rebounding from the ground with initial velocity $v_{2i} = v_0$ (positive upward), and the small ball still falling downward with initial velocity $v_{1i} = -v_0$. Equation 9.15a gives the final velocity of the small ball as

$$v_f = \left(\frac{m-M}{m+M}\right)(-v_0) + \left(\frac{2M}{m+M}\right)v_0 = \left(\frac{3M-m}{m+M}\right)v_0 \approx 3v_0$$

since $M \gg m$. Once v_f is known, the height it rebounds can be readily calculated by using energy conservation (or kinematic Equation 2.11).

EVALUATE Conservation of total mechanical energy requires that $mv_f^2/2 = mgh_f$, so

$$h_f = \frac{v_f^2}{2g} = \frac{(3v_0)^2}{2g} = 9\frac{v_0^2}{2g} = 9h$$

or about nine times the original height.

ASSESS This demonstration, sometimes called a Minski cannon, is striking. Try it with a new tennis ball and properly inflated basketball.

Chapter 10

18. **INTERPRET** In this problem we are given the angular acceleration of a turbine and asked how long it takes to reach its operating speed and the number of revolutions that occur during this start-up period. The key to this type of rotational problem is to understand the analogous situation for linear motion, and apply the appropriate equation The analogies are summarized in Table 10.1.

DEVELOP Given a constant angular acceleration α, the angular velocity and angular position at a later time t can be found using Equations 10.7 and 10.8, respectively:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

The initial and final angular velocities are $\omega_0 = 0$ and

$$\omega = 3600 \text{ rpm} = \frac{3600 \text{ rev}}{1 \text{ min}} = \frac{2\pi(3600) \text{ rad}}{60 \text{ s}} = 377 \text{ rad/s}$$

EVALUATE (a) The amount of time it takes to reach its operating speed is

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{377 \text{ rad/s} - 0}{0.52 \text{ rad/s}^2} = 725 \text{ s} = 12 \text{ min}$$

(b) Using Equation 10.8, we find the number of turns made during this time interval to be

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(0.52 \text{ rad/s}^2\right)\left(725 \text{ s}\right)^2 = 1.37\times 10^5 \text{ rad} = 2.2\times 10^4 \text{ rev}$$

ASSESS The responses are given to two significant figures to reflect the precision of the data. The turbine turns very fast. After 12.1 min, it has reached an angular speed of 377 rad/s, or 60 rev/s!

21. **INTERPRET** This problem involves the concept of torque. We are given the force applied (by the child) and are asked to find the lever arm needed to produce a given torque.

DEVELOP Solve Equation 10.10 for the lever arm r, assuming the child pushes perpendicular to the door (so $\theta = 90°$).

EVALUATE Solving Equation 10.10 for r gives

$$r = \frac{\tau}{F\sin\theta} = \frac{110 \text{ N}\cdot\text{m}}{(90 \text{ N})\sin(90°)} = 1.2 \text{ m}$$

so the child must push 1.2 m from the center of the door.

ASSESS This seems like a reasonable distance, given that the typical width of revolving doors is greater than 1.2 m.

33. **INTERPRET** This problem involves calculating the torque that results from a frictional force applied about a 41-cm shaft, and the angular acceleration this engenders. We are then asked to find the time it takes the shaft (and the accompanying flywheel) to stop, given their initial rotational speed.

DEVELOP From Equation 10.10, the torque applied to the flywheel is

$$\tau = rF\sin\theta = R_{\text{shaft}} f_{\text{k}}$$

where $\theta = 90°$, $f_{\text{k}} = 34$ kN, and $R_{\text{shaft}} = (41 \text{ cm})/2 = 0.205$ m. Inserting this torque into the rotational analog of Newton's second law (for constant mass), we can find the angular acceleration. We find $\alpha = -\tau/I_{\text{fw}}$, where the negative sign indicates that the acceleration is directed opposite to the motion. Use Table 10.2 to find the formulas for the rotational inertia of the flywheel (which we take to be a solid disk). This is

$$I_{\text{fw}} = \frac{1}{2}M_{\text{fw}}R_{\text{fw}}^2$$

where $M_{\text{fw}} = 7.7 \times 10^4$ kg and $R_{\text{fw}} = 2.4$ m. The time it will take the flywheel to stop is, from Equation 10.7 with $\omega = 0$,

$$0 = \omega_0 + \alpha t$$

$$t = -\frac{\omega_0}{\alpha} = \frac{\omega_0 I_{\text{fw}}}{\tau} = \frac{\omega_0 M_{\text{fw}} R_{\text{fw}}^2}{2R_{\text{shaft}} f_{\text{k}}}$$

EVALUATE Inserting the given quantities into the expression for the time gives

$$t = \frac{\omega_0 M_{\text{fw}} R_{\text{fw}}^2}{2 f_{\text{k}} R_{\text{shaft}}} = \frac{(360 \text{ rpm})\left(7.7\times 10^4 \text{ kg}\right)(2.4 \text{ m})^2}{2\left(34\times 10^3 \text{ N}\right)(0.205 \text{ m})}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60 \text{ s}}\right) = 1200 \text{ s} = 20 \text{ min}$$

37. **INTERPRET** We are asked to find the energy stored in a the flywheel of Problem 10.33, so we will use the concepts of rotational inertia and kinetic energy of rotation. We also need to find the power output of a generator if the speed of the flywheel changes a given amount in a given time.

DEVELOP Apply Equation 10.18, $K = I\omega^2/2$, to calculate the kinetic energy stored in the flywheel. We will need to convert the angular speed in rpm to rad/s, and calculate the rotational inertia of the flywheel disk using $I = mR^2/2$ (from Table 10.2). From the work-energy theorem (see Equation 10.19) and using $\overline{P} = W/\Delta t$, we have

$$\overline{P} = \frac{W}{\Delta t} = \frac{\Delta K}{\Delta t}$$

where $\omega_i = 360$ rpm, $\omega_f = 300$ rpm, and $\Delta t = 3$s.

The mass of the flywheel is m = 7.7×10^4 kg, the radius is $R = 2.4$ m, and the initial rotation rate is 360 rpm.

EVALUATE

(a) The energy stored in the flywheel is

$$K = \frac{1}{2}I\omega^2 = \frac{1}{4}mR^2\omega^2$$

$$= \frac{1}{4}(7.7\times10^4\text{ kg})(2.4\text{ m})^2\left(360\,\frac{\text{rev}}{\text{min}}\right)^2\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)^2\left(\frac{1\text{ min}}{60\text{ s}}\right)^2 = 1.6\times10^8\text{ J}$$

(b) The average power output during the deceleration of the flywheel is

$$\overline{P} = \frac{\Delta K}{\Delta t} = \frac{mR^2}{4\Delta t}\left(\omega_f^2 - \omega_i^2\right)$$

$$= \frac{(7.7\times10^4\text{ kg})(2.4\text{ m})^2}{4(3\text{ s})}\left[\left(300\,\frac{\text{rev}}{\text{min}}\right)^2 - \left(360\,\frac{\text{rev}}{\text{min}}\right)^2\right]\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)^2\left(\frac{1\text{ min}}{60\text{ s}}\right)^2 = 16\text{ MW}$$

ASSESS This is a good way of generating enormous power pulses.

40. **INTERPRET** This problem involves rotational kinetic energy and rotational inertia. Knowing the fraction of kinetic energy due to rotation, we are to determine whether the ball is solid or hollow.

DEVELOP The given fraction of kinetic energy due to rotation is

$$f = \frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{40}{100} = \frac{2}{5}$$

We know that $K_{tot} = mv^2/2 + K_{rot}$, and $K_{rot} = I\omega^2/2$. From Table 10.2, we find that the rotational inertia for a hollow sphere is $I = 2MR^2/3$, whereas for a solid sphere it is $I = 2MR^2/5$. Use these formulas to calculate the ratio of rotational to total kinetic energy to see which one corresponds to the ratio give ($f = 2/5$).

EVALUATE For the solid sphere,

$$f = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2} = \frac{\left(\frac{2}{5}MR^2\right)\omega^2}{M(\omega R)^2 + \left(\frac{2}{5}MR^2\right)\omega^2} = \frac{\frac{2}{5}}{1+\frac{2}{5}} = \frac{2}{7}.$$

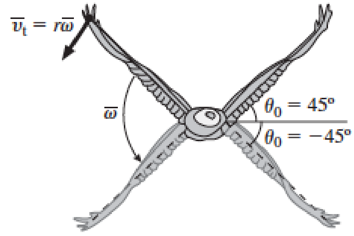
For the hollow sphere,

$$f = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2} = \frac{\left(\frac{2}{3}MR^2\right)\omega^2}{M(\omega R)^2 + \left(\frac{2}{3}MR^2\right)\omega^2} = \frac{\frac{2}{3}}{1+\frac{2}{3}} = \frac{2}{5}$$

Therefore, the sphere must be hollow.

ASSESS Notice that the rotation kinetic energy comprises a larger fraction of the total kinetic energy for a hollow sphere because more of its mass is concentrated away from the axis of rotation, so the rotational inertia is greater.

43. **INTERPRET** We're asked to characterize one of the eagle's downstrokes, in which case the top of the stroke is θ_0 and the bottom of the stroke is θ, as shown in the figure below.



DEVELOP If the eagle flaps 20 times per minute, then it makes a full flap every 3 seconds. A full flap consists of an upstroke and a downstroke, so a single downstroke takes $\Delta t = 1.5$ s. We can plug this time into Equation 10.1 to determine the average angular velocity $(\overline{\omega} = \Delta\theta/\Delta t)$. The tangential velocity at the tip can be found using Equation 10.3, $\overline{v} = \overline{\omega}r$. For the radius, r, we assume it's roughly half the wingspan, which is by definition the distance between the two wing tips.

EVALUATE (a) Let's first convert the angles from degrees to radians:

$$\theta_0 = 45°\left(\tfrac{2\pi\text{ rad}}{360°}\right) = 0.785\text{ rad}\;;\quad \theta = -45°\left(\tfrac{2\pi\text{ rad}}{360°}\right) = -0.785\text{ rad}$$

So the average angular velocity is:

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{(0.785\text{ rad}) - (-0.785\text{ rad})}{1.5\text{ s}} = 1.05\text{ rad/s} \simeq 1.1\text{ rad/s}$$

(b) As for the tangential velocity at the tip of the wing:

$$\overline{v} = \overline{\omega}r = (1.05\text{ rad/s})\left(\tfrac{1}{2}\cdot 2.1\text{ m}\right) = 1.1\text{ m/s}$$

ASSESS The eagle makes a single downstroke in 1.5s, which seems reasonable. And since its wings are about a meter long each, it makes sense that the tangential velocity is approximately one meter per second.

53. **INTERPRET** The problem concerns the cellular motor that drives the flagellum of the *E. coli* bacteria. We are asked to find the force exerted by this motor, given the torque and the radius at which the force is applied.

DEVELOP We're told that the force is applied tangentially, so $\theta = 90°$, and Equation 10.10 reduces to: $\tau = rF$.

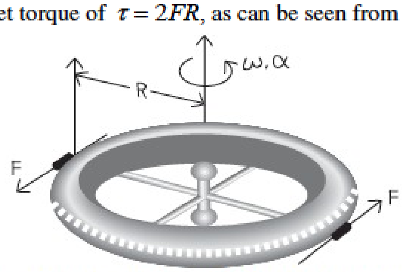
EVALUATE Solving for the motor's applied force:

$$F = \frac{\tau}{r} = \frac{400\text{ pN}\cdot\text{nm}}{12\text{ nm}} = 33\text{ pN}$$

ASSESS This is a very small force, but it's rather impressive that an *E. coli*, with a typical mass of about 10^{-15}kg, can exert a force that is over 1000 times its own weight.

55. **INTERPRET** You are asked to find the time it takes for the space station to start from rest and reach a certain angular speed, with a given thrust.

DEVELOP The space station is essentially a ring with radius $R = 11\text{m}$ and rotational inertia $I = MR^2$ (from Table 10.1). The two rockets provide a net torque of $\tau = 2FR$, as can be seen from the figure below.



This torque causes an angular acceleration, $\alpha = \tau/I = 2F/MR$, that spins up the station from rest to an angular velocity, ω. This final rotation speed is chosen such that the centripetal acceleration at the rim is equal to the gravitational acceleration on the surface of the Earth:

$$a_c = \frac{v^2}{R} = \frac{(\omega R)^2}{R} = \omega^2 R = g \;\rightarrow\; \omega = \sqrt{\frac{g}{R}}$$

Your job is to determine how long the rockets must fire to reach this angular velocity and how many rotations does the station make during this time period.

EVALUATE (a) The time can be found with Equation 10.7:

$$t=\frac{\omega}{\alpha}=\frac{\sqrt{g/R}}{2F/MR}=\frac{M\sqrt{gR}}{2F}=\frac{\left(5.0\times10^5\,\text{kg}\right)\sqrt{\left(9.8\text{ m/s}^2\right)\left(11\text{ m}\right)}}{2\left(100\text{ N}\right)}=2.60\times10^4\,\text{s}=7.2\text{ h}$$

(b) We could use Equation 10.8 to find the number of revolutions completed in this time, but Equation 10.9 provides a simple formula with the weight of the space station:

$$\Delta\theta=\frac{\omega^2}{2\alpha}=\frac{Mg}{4F}=\frac{\left(5.0\times10^5\,\text{kg}\right)\left(9.8\text{ m/s}^2\right)}{4\left(100\text{ N}\right)}=\frac{12,250\text{ rad}}{2\pi\text{ rad/rev}}=1900\text{ rev}$$

ASSESS These are relatively small rockets, so it takes a fair amount of time to reach the desired rotational velocity. Since $t\sim 1/F$, a larger thrust will shorten this spin up time.

58. **INTERPRET** This problem combines Newton's second law for rotational motion and the concept of torque. Combining these with the rotational kinematic equations (Equations 10.6–10.9), we can find the final angular speed of the wheel.

DEVELOP Assuming the wheel spins about an essentially frictionless axis, the only torsional force acting on the wheel is due to the wrench, so Newton's second law (Equation 10.11) gives

$$\tau_{\text{net}}=\tau_{\text{wrench}}=I\alpha$$

From Example 10.6, the rotational inertia of the bicycle wheel is I = MR2, and from Equation 10.10, the torque applied by the wrench is $\tau_{\text{wrench}}=-f_kR=-\mu_kF_{\text{app}}R$. Note that $\theta=90°$ in this case for Equation 10.10 because the frictional force is applied tangentially to the wheel, and we have used Equation 5.3 to express the frictional force. This gives us the angular acceleration, which we can use in Equation 10.7 to find the final angular speed ω.

EVALUATE Inserting the given quantities into the expression derived above using Newton's second law gives

$$\omega=\omega_0+\alpha t=\omega_0+\frac{\tau_{\text{wrench}}}{I}t=\omega_0-\frac{\mu_kF_{\text{app}}R}{MR^2}t=\omega_0-\frac{\mu_kF_{\text{app}}t}{MR}$$

$$=\left(230\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)-\frac{0.46\left(2.7\text{ N}\right)\left(3.1\text{ s}\right)}{\left(1.9\text{ kg}\right)\left(0.33\text{ m}\right)}=18\text{ rad/s}=170\text{ rev/min}$$

ASSESS Notice that the greater the applied force, the smaller will be the final angular momentum. One might think the wheel will reverse direction if the applied force is great enough, but this will not happen because friction only acts to counter the motion, not to create motion. Once the wheel stops, the friction force will be static and will not create motion. (It could, however, prevent another force from turning the wheel.)

77. **INTERPRET** You want to know if a rotating flywheel has as much energy as its manufacturer claims.

DEVELOP The flywheel can be modeled as a ring with rotational inertia $I=MR^2$. Its rotational kinetic energy is $\frac{1}{2}I\omega^2$, from Equation 10.18.

EVALUATE We have to divide the given diameter by 2 to get the radius, and we have to convert the non-SI unit of rpm to rad/s. Following that, the flywheel's kinetic energy is:

$$K_{\text{rot}}=\tfrac{1}{2}MR^2\omega^2=\tfrac{1}{2}\left(48\text{ kg}\right)\left(\tfrac{1}{2}0.39\text{ cm}\right)^2\left(30,000\text{ rpm}\right)^2\left(\frac{\frac{2\pi}{60}\text{ rad/s}}{1\text{ rpm}}\right)^2=9.0\text{ MJ}$$

The specs are incorrect. The flywheel's storage capacity is 3 MJ below what the manufacturer claims.

ASSESS A flywheel is like a battery that stores energy as kinetic rotational energy. It has a high rotational inertia and presumably very little friction, so it will spin freely for a long time without slowing down appreciably. When the need arises, the flywheel can be connected to an electric generator, where its rotational energy is converted to electricity.

Chapter 11

13. **Interpret** This problem is an exercise in determining the direction and magnitude of the angular velocity vector. From the direction and speed at which the car is traveling, we are to deduce the angular velocity of its wheels.

Develop From Chapter 10 (Equation 10.3), we know that the magnitude of the angular velocity (i.e., the angular speed) is given by $\omega = v_{cm}r$. For this problem, we have v_{cm} = (70 km/h)(10^3 m/km)(1 h/3600 s) = 19.44 m/s and r = d/2 = (0.62 m)/2 = 0.31 m. The direction of the angular velocity vector can be determined using the right-hand rule (see Figure 11.1).

Evaluate Inserting the given quantities into Equation 10.3 gives an angular speed of

$$\omega = v_{cm}/r = (19.44\text{ m/s})/(0.31\text{ m}) = 63\text{ s}^{-1}$$

to two significant figures. If the car is rolling noth, the right-hand rule determines that the direction of the anular velocity vector is to the left, which is west. Therefore $\vec{\omega} = 63\text{ s}^{-1}$ west .

Assess Notice that the angular speed may be reported in units of rad/s, but since radians are a dimensionless quantity, they are often left out, leaving s^{-1}, which is a frequency (Hz).

19. **Interpret** You want to know what torque is supplied by the deltoid muscle about the shoulder joint when your arm is outstretched.

Develop From Equation 11.2, the torque is $\vec{\tau} = \vec{r} \times \vec{F}$, with the magnitude equaling $rF\sin\theta$.

Evaluate The distance between the shoulder joint (i.e., where the arm pivots) and where the deltoid force is applied is given as $r = 18$ cm. The angle between the corresponding radial vector and the muscle force is $\theta = 180^\circ - 15^\circ = 165^\circ$. The magnitude of the torque is then

$$\tau = rF\sin\theta = (0.18\text{ m})(67\text{ N})\sin 165^\circ = 3.1\text{ N}\cdot\text{m}$$

By the right-hand rule, we start with our fingers pointing to the right in the direction of $\vec{r}$, and then rotate them upwards in the direction of $\vec{F}$. Our thumb points up, so the torque of $3.1\text{ N}\cdot\text{m}$ points out of the page.

Assess Is this enough torque to keep the arm outstretched? Let's assume the arm has a mass of about 3 kg (corresponding to a weight of about 30 N), and its center of mass is 30 cm from the shoulder joint. The gravitational force will pull the arm down at 90° to the horizontal arm direction, thus generating a torque in the opposite direction with a magnitude of $\tau \simeq (30\text{ N})(0.3\text{ m}) = 9\text{ N}\cdot\text{m}$. Therefore, the deltoid muscle would need help from other muscles to keep the arm horizontal.

22. **Interpret** For this problem, we are to find the angular speed given the angular momentum and the rotational inertia of an object.

Develop Use Equation 11.4, $\vec{L} = I\vec{\omega}$ to find the angular speed of the gymnast.

Evaluate Given that $L = 470\text{ kg}\cdot\text{m}^2\cdot\text{s}^{-1}$ and the $I = 62\text{ kg}\cdot\text{m}^2$, the angular speed of the gymnast must be

$$\omega = \frac{L}{I} = \frac{470\text{ kg}\cdot\text{m}^2\cdot\text{s}^{-1}}{62\text{ kg}\cdot\text{m}^2} = 7.6\text{ s}^{-1}$$

Assess The angular speed has units of frequency, as expected. It may equivalently be expressed as 7.6 rad/s, because radians are dimensionless.

25. **INTERPRET** This problem involves conservation of angular momentum, which we can use to find the angular speed of a spinning wheel after a piece of clay is dropped onto it and sticks to its surface.

DEVELOP If the clay is dropped vertically onto a horizontally spinning wheel, the angular momentum about the vertical spin axis is conserved. Conservation of angular momentum is expressed as

$$\vec{L}_i = \vec{L}_f \quad \Rightarrow \quad I_i\vec{\omega}_i = I_f\vec{\omega}_f$$

For this problem, the direction of the angular velocity does not change, so this expression for conservation of angular momentum reduces to its scalar form, $I_i\omega_i = I_f\omega_f$. The intial rotational inertia is $I_i = I_{wheel} = 6.40$ kg·m^2·s^{-1}, and the final rotational inertia is $I_f = I_{wheel} + m_{clay}r^2$.

EVALUATE Inserting the given quantities into the expression from conservation of angular momentum, the final angular velocity is

$$\omega_f = \frac{I_i}{I_f}\omega_i = \left(\frac{I_{wheel}}{I_{wheel} + m_{clay}r^2}\right)\omega_i = \frac{6.40\ \text{kg·m}^2}{6.40\ \text{kg·m}^2 + (2.70\ \text{kg})(0.460\ \text{m})^2}(19.0\ \text{rpm}) = 17.4\ \text{rpm}$$

ASSESS The clay increases the total rotational inertia of the system, so the angular speed decreases, as required by conservation of angular momentum.

35. **INTERPRET** We need to find the angular momentum of a disk-shaped rotor that is part of a micromechanical device that measures blood flow.

DEVELOP The angular momentum of the rotor is $L = I\omega$, where the rotational inertia is that of a disk: $I = \frac{1}{2}MR^2$. We don't explicitly know the rotor's mass, but the material is silicon, which has a density of $\rho = 2.33\ \text{g/cm}^3$.

EVALUATE The mass of the rotor is the density times the volume: $M = \rho\left(d\cdot\pi R^2\right)$, where d is the rotor's thickness. The radius is half the diameter: $R = 150\mu m$, and the 800-rpm rotational speed converted to SI units is: $\omega = 83.8$ rad/s. So the angular momentum of the rotor during the tests is

$$L = I\omega = \frac{\pi}{2}\rho d R^4\omega$$
$$= \frac{\pi}{2}\left(2.33\times10^3\text{kg/m}^3\right)\left(2.0\times10^{-6}\text{m}\right)\left(150\times10^{-6}\text{m}\right)^4(83.8\ \text{rad/s}) = 3.1\times10^{-16}\text{J}\cdot\text{s}$$

ASSESS This is a very small angular momentum, but we expect it to be. Otherwise, the device would significantly disturb the blood flow it is designed to measure.

41. **INTERPRET** This problem is about the rotational motion of the skaters, given their initial linear speed and radius of the circle they traverse. The aim is to keep the final linear speed and centripetal force below the stated maximums. The key concept here is conservation of angular momentum.

DEVELOP If the ice is frictionless, the only external force on the skaters is the force that brings the end-skater to a sudden stop at a point we'll call *P*. (Note: The forces they exert on each other through their hands are internal forces.) The stopping force exerts no torque about point *P*, so the total angular momentum about a vertical axis through *P* is conserved. Initially, the other seven skaters are each moving with the same linear momentum $(p = mv_0)$ in a direction perpendicular to the line that connects them $(\sin\theta = 1)$. So from Equation 11.3, the angular momentum of each skater about *P* is

$$L_{0n} = |\vec{r}_n \times \vec{p}_n| = r_n(mv_0)\sin\theta = mv_0r_n$$

where r_n is the distance between the *n*-th skater and the point *P*: $r_n = n(\ell/7)$ for $n = 1, 2, \ldots, 7$ and $\ell = 12$ m. The total initial angular momentum is the sum $L_0 = \sum_{n=1}^{7} L_{0n}$, which will be conserved when the group starts rotating and has an angular momentum of $L_f = I\omega$. Here, the rotational inertia is $I = \sum_{n=1}^{7} mr_n^2$. From all this we can determine the rotational speed, which will give us the linear speed and centripetal force on the outside skater $(n = 7)$.

Evaluate The total initial angular momentum is

$$L_0 = \sum_{n=1}^{7} m v_0 r_n = \frac{m v_0 \ell}{7} \sum_{n=1}^{7} n = \frac{m v_0 \ell}{7}\left[\frac{7\times 8}{2}\right] = 4 m v_0 \ell$$

where we have used $\sum_{n=1}^{N} n = N(N+1)/2$. Similarly, the rotational inertia of the 7 skaters around point P is

$$I = \sum_{n=1}^{7} m r_n^2 = \frac{m\ell^2}{49} \sum_{n=1}^{7} n^2 = \frac{m\ell^2}{49}\left[\frac{7\times 8\times 15}{6}\right] = \frac{20 m\ell^2}{7}$$

where we have used $\sum_{n=1}^{N} n^2 = N(N+1)(2N+1)/6$. Since angular momentum is conserved $(L_0 = L_f)$, we can solve for the angular speed:

$$\omega = \frac{L_0}{I} = \frac{4 m v_0 \ell}{\frac{20}{7} m\ell^2} = \frac{7 v_0}{5\ell}$$

The outside skater will have a tangential speed of $v = \omega\ell$, so in order to keep this below 8.0 m/s, the initial speed can't exceed:

$$v_0 = \tfrac{5}{7} v < \tfrac{5}{7}(8.0\ \text{m/s}) = 5.7\ \text{m/s}$$

The force on the outside skater's hand is the centripetal force: $F = ma_c = m\ell\omega^2$. To keep This below 300 N, the initial speed can't exceed:

$$v_0 = \frac{5}{7}\sqrt{\frac{F\ell}{m}} < \frac{5}{7}\sqrt{\frac{(300\ \text{N})(12\ \text{m})}{(60\ \text{kg})}} = 5.5\ \text{m/s}$$

This limit is stricter than the one above. The greatest speed that the skaters can go before the rotational maneuver is 5.5 m/s.

Assess Notice that the outside skater will be going 1.4 times faster following the maneuver. By contrast, the skaters closer to the point P will slow down after the maneuver $(v_n = \omega r_n)$. This makes sense: to keep the total angular momentum constant, some skaters will gain angular momentum, while others will lose it.

48. **Interpret** To increase the surface area of this alien planet, you plan to hollow out its center. This will increase the rotational inertia, so to conserve angular momentum, the planet's rotation will slow down.

Develop The planet is originally a solid sphere of radius R_0. When the planet is hollowed out, the radius of its outer surface is R, and the radius of its inner surface is $\frac{4}{5}R$, such that the shell thickness is $\frac{1}{5}R$. No material is added or taken away during this alteration, so the total mass, $M = \int dm$, should remain constant. To calculate the mass, divide the planet up into concentric shells of infinitesimal thickness. For a given shell of radius R', the mass is $dm = \rho\cdot 4\pi R'^2 dR'$, where ρ is the planet's density and $4\pi R'^2 dR'$ is the volume of the given shell. For the original planet, R' varies from 0 to R_0, while for the hollowed out planet, R' varies from $\frac{4}{5}R$ to R. Equating the mass integrals for the two cases gives:

$$\int_0^{R_0} 4\pi\rho R'^2 dR' = \int_{4R/5}^{R} 4\pi\rho R'^2 dR' \quad\rightarrow\quad R = \frac{5R_0}{\sqrt[3]{5^3 - 4^3}} \simeq 1.27 R_0$$

This can be used to find the increase in surface area. But to find the change in the length of the day, you have to find the change in the rotational inertia of the planet. The original sphere has $I_0 = \frac{2}{5}MR_0^2$, from Table 10.1. However, the formula for a hollow sphere in Table 10.1, $I = \frac{2}{3}MR^2$, assumes the shell is thin, which is not the case here. What you can do is sum over the infinitesimal shells with mass dm defined above. Each of them has rotational inertia of:

$$dI = \frac{2}{3}(dm)R'^2 = \frac{8\pi}{3}\rho R'^4 dR' = \frac{2M}{R_0^3} R'^4 dR'$$

where the mass relation for the original sphere was used: $M = \rho\left(\frac{4\pi}{3}R_0^3\right)$. Integrating over all the shells in the hollowed sphere gives:

$$I = \int dI = \frac{2M}{R_0^3}\int_{4R/5}^{R} R'^4 dR' = \frac{2MR^5}{5R_0^3}\left[1-\left(\tfrac{4}{5}\right)^5\right]$$

Notice that if R' varies from 0 to R_0, as in the original case, the integration returns the familiar result of $I = \frac{2}{5}MR_0^2$.

Evaluate By hollowing out the planet, the surface area increases by

$$\frac{A}{A_0} = \frac{4\pi R^2}{4\pi R_0^2} = \left(\frac{R}{R_0}\right)^2 = 1.27^2 = 1.61$$

The angular momentum is conserved, so $I_0\omega_0 = I\omega$. The period is inversely proportional to rotation speed $(T = 2\pi/\omega)$, so the length of the day will increase by

$$\frac{T}{T_0} = \frac{\omega_0}{\omega} = \frac{I}{I_0} = \frac{\frac{2}{5}MR^5/R_0^3}{\frac{2}{5}MR_0^2}\left[1-\left(\tfrac{4}{5}\right)^5\right] = \left(\frac{R}{R_0}\right)^5\left[1-\left(\tfrac{4}{5}\right)^5\right] = 2.22$$

Assess Suppose the hollowed sphere has thickness Δ, where $\Delta \ll R$. Then, the radii are related by $R \approx R_0/\sqrt[3]{3\Delta}$. Substituting this into the rotational inertia equation, and making a further approximation, gives

$$I = \frac{2MR^5}{5R_0^3}\left[1-(1-\Delta)^5\right] \approx \tfrac{2}{5}MR^2\left(\frac{R}{R_0}\right)^3[5\Delta] = \tfrac{2}{3}MR^2$$

This is the formula for a hollow sphere given in Table 10.1, which shows that this expression only becomes valid when the thickness of the shell is much less than the radius.

54. **Interpret** We use conservation of angular momentum to find the radius of a white dwarf star. We know the initial radius, mass, and rotational speed; so this gives us the initial angular momentum. The final angular momentum will be the same, so we use it to find the radius knowing the final mass and angular speed.

Develop We're told that the star collapses with 60% of its original mass. That means 40% of the mass is "blown off." We'll assume these outer layers take their angular momentum with it. So we'll only deal with conservation of momentum in the star's core with $M = 0.6\,M_{sun}$. Assuming the star is uniform, this core initially occupies a sphere with radius:

$$R_0 = \sqrt[3]{\frac{M}{\frac{4\pi}{3}\rho_{sun}}} = \sqrt[3]{\frac{0.6M_{sun}}{\frac{4\pi}{3}\left(M_{sun}/\frac{4\pi}{3}R_{sun}\right)}} = \sqrt[3]{0.6}R_{sun}$$

Evaluate Before the collapse, the core's angular momentum is given by $L = I_0\omega_0$, where $I_0 = \frac{2}{5}MR_0^2$, and $\omega_0 = 2\pi/25\text{ d}$. After the collapse, the core still has the same angular momentum, but the expression is now $L = I\omega$, where $I = \frac{2}{5}MR^2$, and $\omega = 2\pi/131\text{ s}$. Solving for the unknown final radius, we get:

$$R = R_0\sqrt{\frac{\omega_0}{\omega}} = \left(\sqrt[3]{0.6}R_{sun}\right)\sqrt{\frac{131\text{ s}}{25\cdot 24\cdot 3600\text{ s}}} = 6.57\times10^{-3}R_{sun} = 4.57\times10^6\text{ m}$$

This radius is about 70% of the radius of the Earth, and 150 times smaller than the original star.

Assess One could assume that the outer layers blow off without taking away any of the angular momentum, and the core inherits *all* of the original angular momentum of the star before the collapse: $L = \frac{2}{5}M_{sun}R_{sun}^2\omega_0$. (This is unlikely but it can serve as an upper bound.) In such a case, the final radius would be 100 times smaller than the original star.