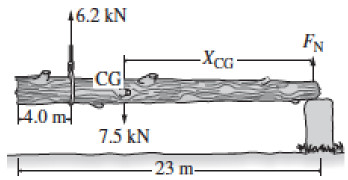
Homework #12

19. **INTERPRET** The log is in equilibrium under the torques exerted by the cable, gravity, and the wall.

DEVELOP Because the tree is in equilibrium, we know that the sum of the torques is zero. Since we don't know the force exerted by the wall at the right end of the tree, we choose the contact point with the wall as our pivot point, so that the torque from this force is zero. As for the given forces, gravity acts at the center of gravity, producing a torque of $\tau_g = x_{CG}F_g$, where x_{CG} is the unknown distance between the wall and the center of gravity. See the figure below. The cable's tension produces a torque in the opposite direction: $\tau_c = -x_c T$, where $x_c = 23\text{m} - 4.0\text{m} = 19\text{m}$.



EVALUATE Setting the sum of the torques to zero gives $\tau_g = -\tau_c$, from which we can find the center of gravity:

$$x_{CG} = \frac{x_c T}{F_g} = \frac{(19\text{m})(6.2\text{ kN})}{(7.5\text{ kN})} = 16\text{ m}$$

where this is relative to the wall.

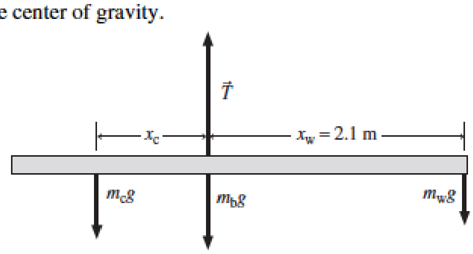
ASSESS This seems reasonable. As expected, the center of gravity is between the cable and the wall, otherwise the tree couldn't possibly be in equilibrium.

22. **INTERPRET** This problem is about static equilibrium. We want to know where the concrete should be placed so that the long beam suspended by a cable and with a steelworker standing at one end will be in equilibrium.

DEVELOP Make a sketch of the situation, showing all the forces and the positions at which they are applied (see figure below). Because the beam is uniform, its center of mass is at its geometric center, which is where the cable is attached and where we can consider gravity to act (i.e., its center of gravity). At equilibrium, the sum of the torques on the beam about any point will be zero. We choose to calculate the torque about the center of gravity (which is the same as the center of gravity for a uniform gravitational field), so that the tension of the cable and the force of gravity on the beam itself do not enter (because they act at a distance zero from the chosen pivot point). Therefore,

$$(\Sigma\tau)_{cm} = 0 = -m_w g x_w + m_s g x_s$$

where the negative sign enters because the worker is to the left of the center of gravity, which we choose to be the negative direction. The quantities m_c and m_w are the masses of the concrete and the worker, and $x_s = 2.1$ m is the distance from the worker to the center of gravity.



EVALUATE Using the values given in the problem statement, we have

$$x = \left(\frac{m_w}{m_c}\right)x_s = \left(\frac{65\text{ kg}}{190\text{ kg}}\right)(2.1\text{ m}) = 0.72\text{ m}$$

to the right of the center of gravity.

24. **INTERPRET** The problem is about the stability of the roller coaster as it moves along the track described by a height function. We want to identify the equilibrium point and classify its stability.

DEVELOP The potential energy of the roller coaster car is

$$U(x) = mgh(x) = mg\left(0.94x - 0.01x^2\right)$$

The equilibrium condition is given by Equation 12.3 $dU/dt = 0$. In addition, the equilibrium condition may be classified according to its second derivative:

$$\frac{d^2U}{dx^2}\begin{cases} > 0, & \text{stable} \\ < 0, & \text{unstable} \\ = 0, & \text{neutral} \end{cases}$$

EVALUATE **(a)** Applying Equation 12.3, the condition for equilibrium, gives

$$0 = \frac{dU}{dx} = mg\left(0.94 - 0.02x\right) \quad \Rightarrow \quad x = 47\text{ m}$$

Thus, at 47 m from the origin, the roller coaster will be in an equilibrium position.

(b) Using Equation 12.5, the condition for stable equilibrium, gives

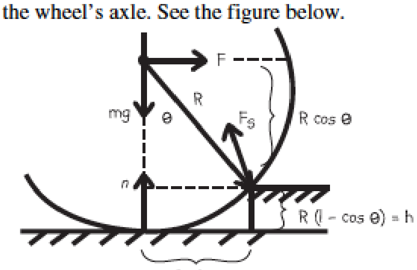
$$\frac{d^2U}{dx^2} = -0.02mg < 0$$

so this is an unstable equilibrium (i.e., a peak, not a valley).

ASSESS The point $x = 47$ m with $U(47\text{ m}) = 22.1mg$ corresponds to a local maximum where the potential-energy curve is concaving downward. Therefore, the point is unstable.

29. **INTERPRET** You need to specify the minimum horizontal force needed to push the cart over the step. The cart will be in static equilibrium up until the force is sufficient to overcome the obstacle.

DEVELOP The forces that you need to account for are: the force, F, exerted by the person; the weight of the cart, mg; the normal force, n, between the wheel and the ground, and the force from the step, F_s. You can assume that the person is only pushing and not lifting the cart in any way. In any case, the person's force and cart's weight are both exerted on the wheel at the wheel's axle. See the figure below.



Since you don't know the magnitude or the direction of the step force, you should choose as the pivot point where the wheel meets the step. Before the cart starts moving, the wheel is in static equilibrium, so the sum of the torques around this pivot point should be zero:

$$\left(\Sigma\tau\right)_{\text{step}} = mgR\sin\theta - nR\sin\theta - FR\cos\theta = 0$$

where we have used the trig identity: $\sin\left(90° - \theta\right) = \cos\theta$. As the person pushes harder, more of the cart's weight shifts from the ground (supported by n) to the step (supported by F_s). Eventually, the normal force will go to zero, and the wheel will rotate around the point where it meets the step. Therefore, the minimum force needed to push the cart over the step is that which makes $n = 0$:

$$mgR\sin\theta - FR\cos\theta = 0 \quad \rightarrow \quad F = mg\tan\theta$$

EVALUATE To find the minimum force, we need to find the angle θ. We know that the step height is equal to: $h = R\left(1 - \cos\theta\right)$. So

$$\theta = \cos^{-1}\left[1 - \frac{h}{R}\right] = \cos^{-1}\left[1 - \frac{8\text{ cm}}{\frac{1}{2}\cdot 60\text{ cm}}\right] = 42.83°$$

Plugging this into the force equation from above:

$$F = mg\tan\theta = \left(55\text{ kg}\right)\left(9.8\text{ m/s}^2\right)\tan\left(42.83°\right) = 500\text{ N}$$

27. **INTERPRET** This problem involves finding the torque about a pivot point given the forces and the positions at which they are applied. This is also a problem concerning equilibrium, in which the net force and torque on an object must be zero. We can use this to find the force required of the deltoid muscle to keep the arm in equilibrium.

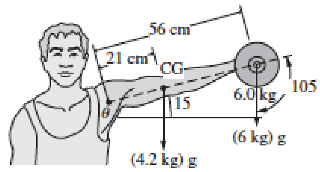
DEVELOP The figure below shows the forces involved and where they are applied. To find the torque about the shoulder due to the arm and the 6-kg mass, we sum the torques. This gives

$$\tau_a = r_{cm} m_{arm} g \sin\theta + r_{arm} m g \sin\theta$$

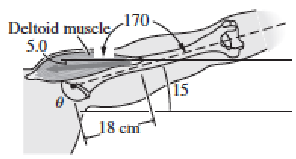
where $r_{cm} = 21$ cm, $r_{arm} = 56$ cm , $m_{arm} = 4.2$ kg, $m = 6.0$ kg, and $\theta = 105°$. To find the tensile force required of the deltoid muscle, we require that all the torques (including now the torque due to the deltoid muscle) sum to zero (this is the condition for equilibrium, see Equation 12.2). This gives

$$\left(\sum\tau\right)_{shoulder} = 0 = r_d T \sin\theta_d + r_{cm} m_{arm} g \sin\theta + r_{arm} m g \sin\theta = r_d T \sin\theta_d + \tau_a$$

where $\theta_d = 170°$ and $r_d = 18$ cm. We can solve this for the tensile force T of the deltoid muscle.



(a)



(b)

EVALUATE (a) The torque due to the arm and the mass is

$$\tau_a = r_{cm} m_{arm} g \sin\theta + r_{arm} m g \sin\theta = \left[(0.21\ \text{m/s})(4.2\ \text{kg}) + (0.56\ \text{m})(6.0\ \text{kg})\right](9.8\ \text{m/s}^2)\sin(255°) = 4.0\times10^1\ \text{N}\cdot\text{m}$$

to two significant figures.

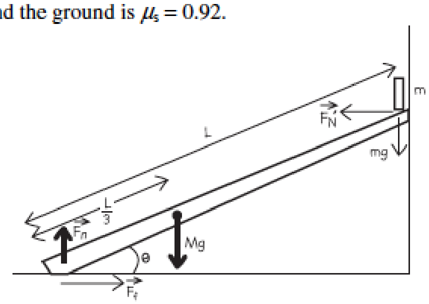
(b) The magnitude of the tensile force supplied by the deltoid is thus

$$T = \frac{\tau_a}{r_d \sin\theta_d} = \frac{40.2\ \text{N}\cdot\text{m}}{(0.18\ \text{m})\sin(170°)} = 1.3\ \text{kN}$$

ASSESS By the right-hand rule, we see that the torque supplied by the deltoid muscle is out of the page, whereas the torque due to the arm and the mass is into the page. Thus, the deltoid muscle must supply a force that is (1.3 kN)/[(9.8 m/s2)(6.0 kg)] = 22 times weight in order to lift it. This is not very efficient, and it underscores the comment at the end of Example 12.3 that the skeleto-muscular structure of the human extremities evolved for speed and range of motion, not mechanical advantage.

37. **INTERPRET** This problem is an equilibrium problem, as long as the log does not slip! We use the conditions for equilibrium to find the maximum allowable mass for a climber standing at the upper end of the log.

DEVELOP We start by drawing a free-body diagram showing all of the forces on the log, as shown in the figure below. The maximum frictional force on the lower end of the log is $f = \mu_s n_1$. Because the log is in equilibrium, this frictional force must be balanced by the normal force n_w from the wall (see Equation 12.1), so if the force from the wall is *greater* than the maximum frictional force, then the log will slip. We can find the force due to the wall by using Equation 12.2, $\sum\tau = 0$ with the lower end of the log as the pivot point. Because the vertical forces must sum to zero (Equation 12.1), we see from the figure that $n_1 = (M+m)g$, where $M = 340$ kg is the mass of the log, and m is the unknown mass of the climber. The length of the log is $L = 6.3$ m and the log forms an angle $\theta = 27°$ with the horizontal. The center of mass of the log is located $L/3$ from the left end, and the coefficient of friction between the left end of the log and the ground is $\mu_s = 0.92$.



EVALUATE From the torque condition for equilibrium, we find

$$\sum \vec{\tau} = 0 = mgL\cos\theta + \frac{MgL}{3}\cos\theta - n_w L\sin\theta$$
$$n_w = (m + M/3)\,g\cot\theta$$

From the equilibrium equation for the horizontal forces, we find

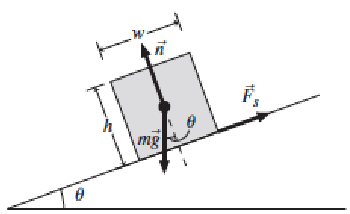
$$f = n_w$$
$$\mu n_1 = \mu(m+M)\,g = (m+M/3)\,g\cot\theta$$
$$\mu m + \mu M = \frac{m}{\tan\theta} + \frac{M}{3\tan\theta}$$
$$m\left(\mu - \frac{1}{\tan\theta}\right) = M\left(\frac{1}{3\tan\theta} - \mu\right)$$
$$m = M\left(\frac{\mu - \cot\theta}{\frac{1}{3}\cot\theta - \mu}\right) = M\left(\frac{3(0.92)\tan(27^\circ) - 3}{1 - 3\tan(27^\circ)}\right) = 3.0M \;=1.0\times10^3\,\text{kg}$$

41. **INTERPRET** In this problem a cubical block is placed on an incline. Given the coefficient of friction between the block and the incline, we'd like to find out whether the block first slides or tips when the angle of the incline is increased. The condition for tipping forward is that the sum of the counter-clockwise torques about the leading corner becomes nonzero. The condition for sliding is that the sum of the forces parallel to the incline becomes nonzero.

DEVELOP We suppose that the block is oriented with two sides parallel to the direction of the incline, and that its center of mass is at the center. Make a sketch of the situation that shows all the forces and the positions at which they act (see figure below). The condition for sliding is that the sum of the forces parallel to the incline becomes nonzero. Taking the direction down the incline to be positive, this condition gives

$$mg\sin\theta_{\text{slide}} - f_s^{\max} > 0$$
$$mg\sin\theta_{\text{slide}} > \mu_s n = \mu_s mg\cos\theta$$
$$\tan\theta_{\text{slide}} > \mu_s$$

where we have used Equation 5. 2 for static friction ($f_s \le \mu_s n$) and we have applied Newton's second law ($F = ma$) in the direction normal to the incline to find $mg\cos\theta_{\text{slide}} = n$. The condition for tipping over is that the center of mass lies to the left of the lower corner of the block (see sketch), or $\theta_{\text{tip}} > 45°$. Compare θ_{tip} to θ_{slide} to see if the cube will slide first or tip.



EVALUATE The block will slide at the angle

$$\theta_{\text{slide}} = \text{atan}(\mu_s) = \text{atan}(0.95) = 43.5° < \theta_{\text{tip}}$$

Thus, the cube slides before tipping.

ASSESS For a general rectangular solid, the angle at which the block will tip is θ = atan (h/w), where w is the width and h is the height of the object. We find that sliding happens first if $\mu_s < w/h$. This makes sense because when the coefficient of friction is small, the block has a greater tendency to slide. On the other hand, when the coefficient of friction is large ($\mu_s > w/h$), we'd expect tipping to take place first. Another way to understand this is to consider a given coefficient of static friction, and let the width w decrease. This will make the block tip because the ratio w/h will decrease to less than μ_s, as expected for the geometry of a tall, thin block placed on an incline.

43. INTERPRET In this problem a ladder is leaning against the wall and we want to find the mass of the heaviest person who can climb to the top of the ladder while keeping it in static equilibrium. We will therefore apply the conditions for static equilibrium; namely, that the sums of the forces and torques on the ladder are zero.

DEVELOP The forces on the uniform ladder are shown in the sketch below, with the force exerted by the (frictionless) wall being horizontal. Consider a person who has climbed up the ladder a fraction α of its length. Equilibrium conditions (Equations 12.1 and 12.2) require that

$$\begin{aligned}
0 &= \sum F_x = f - F_{\text{wall}} \\
0 &= \sum F_y = n - (m_L + m)g \\
0 &= \left(\sum \tau\right)_A = F_{\text{wall}} L \overbrace{\sin(\pi-\theta)}^{=\cos\theta} + \left(\frac{L}{2}\right) m_L g \overbrace{\sin(\theta+\pi/2)}^{=-\sin\theta} + mg\alpha L \overbrace{\sin(\theta+\pi/2)}^{=-\sin\theta} \\
&= F_{\text{wall}} L\cos\theta - \left(\frac{L}{2}\right) m_L g\sin\theta - mg\alpha L\sin\theta
\end{aligned}$$

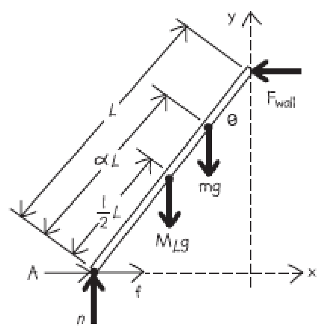
The ladder will not slip if $f \le \mu_s n$. Using the equations above, this condition can be rewritten as

$$f = F_{\text{wall}} = \left(\frac{1}{2}m_L + \alpha m\right) g\tan\theta \le \mu_s n = \mu_s (m_L + m) g$$

or

$$\alpha \le \frac{\mu_s (m_L + m)\cot\theta - m_L/2}{m} = \mu_s \cot\theta + \frac{m_L}{m}\left(\mu_s\cot\theta - \frac{1}{2}\right)$$

Here, we used the horizontal force equation to find f, the torque equation to find F_{wall}, and the vertical force equation to find n.



EVALUATE For a person at the top of the ladder, $\alpha = 1$, and the condition for no slipping becomes

$$m \le m_L \left(\frac{\mu_s\cot\theta - 1/2}{1 - \mu_s\cot\theta}\right)$$

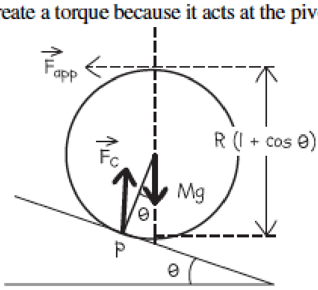
With the data given for the ladder [note that $\cot\theta = \cot(\pi/2 - 66°) = \tan(66°)$], we obtain

$$m \le (9.5\text{ kg})\frac{(0.42)\tan(66°) - 1/2}{1-(0.42)\tan(66°)} = 74\text{ kg}$$

ASSESS The above equation shows that when the coefficient of friction becomes too small, $\mu_s\cot\theta < 1/2$, or $\mu_s < \tan\theta/2$ (see Example 12.2), slipping will occur and it's no longer possible for the ladder to remain in static equilibrium. In this situation, nobody can climb up to the top of the ladder without making the ladder slip, regardless of his or her mass.

49. **INTERPRET** In this problem a wheel has been placed on a slope. We want to apply a horizontal force at its highest point to keep it from rolling down. We will apply the conditions for static equilibrium to solve this problem; namely that the sum of the forces on the wheel must be zero and the sum of the torque on the wheel must be zero.

DEVELOP Consider the conditions for static equilibrium of the wheel, under the action of the forces shown in the sketch below. Here F_{app} is the applied horizontal force, F_c is the contact force of the incline (normal plus friction), and we assume that the center of mass is at the geometric center of the wheel. For the wheel to remain in static equilibrium, the forces must satisfy Equation 12.1. Our plan is to compute the torques about P using Equation 12.2. Note that the contact force, F_c does not create a torque because it acts at the pivot point so it has no lever arm.



EVALUATE The torques about the point of contact sum to zero, or

$$0 = (\Sigma\,\tau)_P = F_{app}R(1+\cos\theta) - MgR\sin\theta$$

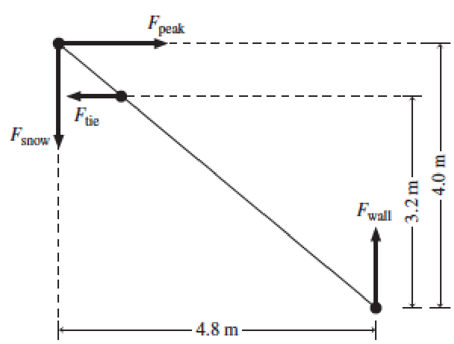
Therefore, the applied force is

$$F_{app} = Mg\frac{\sin\theta}{1+\cos\theta} = Mg\tan\left(\frac{\theta}{2}\right)$$

ASSESS The applied force vanishes when $\theta = 0$ (flat surface), and becomes maximum when $\theta = 90°$. In this limit, $F_{app} = Mg$ and points vertically upward.

63. **INTERPRET** You need to determine the forces on a roof in the event of a snowfall. You use equilibrium techniques, with the sum of torques equaling zero, to determine whether the tie beam will hold.

DEVELOP The figure below shows the relevant forces acting on the rafter located on the right-side of the roof (the forces will be the same on the left-side due to the roof's symmetry). You are told to assume that the contact force, F_{snow}, coming from the weight of the snow and building material is concentrated at the peak. This downward force is countered by the upward force coming from the wall, F_{wall}. The only horizontal forces are from the tie beam, F_{tie}, and from the contact force between the rafters at the peak, F_{peak}. Because the sum of the horizontal forces is zero, F_{tie} will be equal and opposite to F_{peak}.



To find the value of F_{tie}, you'll need to consider the torques on the rafter. The most convenient choice for a pivot point is the joint where the rafter meets the wall. The sum of the torques around this point is

$$\sum \tau = F_{peak}(4.0\text{ m}) - F_{tie}(3.2\text{ m}) - F_{snow}(4.8\text{ m}) = 0$$

EVALUATE Using the fact that $F_{tie} = F_{peak}$, the torque equation gives:

$$F_{tie} = F_{snow}\frac{4.8\text{ m}}{0.8\text{ m}} = (170\text{ kg})(9.8\text{ m/s}^2)(6) = 10\text{ kN}$$

Since this is greater than the limit of 7.5 kN, the tie beam will not hold. As for what forces act on the tie beam, recall that the inward-pointing force, F_{tie}, in the figure is the force *from* the tie beam on the rafter. By Newton's third law, the force *on* the tie beam from the rafter will be in the opposite direction. As such, the tie beam will be pulled outward by each rafter, resulting in tension rather than compression.

ASSESS One way to improve the design would be to choose a longer tie beam that could be placed farther down from the peak. In fact, if the tie beam were 1.1 m, rather than 0.8 m, below the peak in the roof, the tie beam would be able to hold under the weight of the snow.

Chapter 13

16. Interpret This problem deals with the two quantities that characterize periodic motion: period and frequency, which are related by Equation 13.1, $f = 1/T$.

Develop The doctor counts 77 beats per minute. Convert this to a number of beats per second to find the frequency in Hz, and use Equation 13.1 to find the corresponding period in s.

Evaluate The frequency in Hz is

$$f = (77\ \text{bpm})\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 1.3\ \text{Hz}$$

The period T is

$$T = 1/f = 1/(1.28\ \text{Hz}) = 0.78\ \text{s}$$

Assess Frequency and period are reciprocals, so their units are reciprocal as well. For frequency, the units are inverse time ($\text{Hz} = \text{s}^{-1}$) and for the period, the units are time (s).

17. Interpret The question here is about the oscillatory behavior of the violin string. Given the frequency of oscillation, we are asked to find the period.

Develop The relationship between period and frequency is given by Equation 13.1, $T = 1/f$.

Evaluate Using Equation 13.1, we obtain

$$T = \frac{1}{f} = \frac{1}{440\ \text{Hz}} = 2.27\times 10^{-3}\ \text{s}$$

Assess The period is the oscillation is the inverse of the frequency. Note that the unit of frequency is the hertz; (1 Hz = 1 s¯1).

24. Interpret This problem involves finding the maximum velocity and acceleration of a simple harmonic oscillator, given its frequency and maximum displacement.

Develop The displacement of a simple harmonic oscillator can be described by Equation 13.8, $x(t) = A\cos(\omega t + \phi)$. For this problem, we are given $A = 100$ nm, $\omega = 2\pi f$ with $f = 32{,}768$ Hz, and the phase we can take to be $\phi = 0$ without loss of generality. The velocity is the temporal derivative (i.e., Equation 13.9), $v(t) = -\omega A\sin(\omega t)$ (given that $\phi = 0$ for our problem). The acceleration is the derivative of the velocity, or $a(t) = -\omega^2 A\cos(\omega t)$. From these expressions and the given values, we can find the maximum velocity and acceleration.

Evaluate The maximum velocity occurs for $\omega t = (2n+1)\pi/2$, where n is an integer, which gives

$$v_{\text{max}} = \omega A = 2\pi f A = 2\pi(32{,}768)\left(100\times 10^{-9}\ \text{m}\right) = 2.06\ \text{cm/s}$$

The maximum acceleration occurs for $\omega t = n\pi$, where n is an integer, which gives

$$a_{\text{max}} = A\omega^2 = A(2\pi f)^2 = \left(100\times 10^{-9}\ \text{m}\right)\left(2\pi\times 32{,}768\ \text{s}^{-1}\right)^2 = 4.24\ \text{km/s}^2$$

Assess The maximum acceleration and velocity do not occur at the same time, but are out of phase by $\pi/2$, which is one-quarter cycle.

29. **INTERPRET** This problem is about the simple harmonic motion of the pendulum in a grandfather clock. We want to find the time interval between successive ticks.

DEVELOP The period of a simple pendulum is given by Equation 13.15:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

We note that the clock ticks twice each period of oscillation, so the time between clicks is a half-period.

EVALUATE Using Equation 13.15, the time between ticks is

$$\Delta t = \frac{T}{2} = \pi\sqrt{\frac{L}{g}} = \pi\sqrt{\frac{1.45 \text{ m}}{9.81 \text{ m/s}^2}} = 1.21 \text{ s}$$

ASSESS One tick every 1.21 s seems reasonable. Note that if we increase the length of the pendulum, then the period and the time between ticks will increase as well.

32. **INTERPRET** We look at one component of circular motion as simple harmonic motion, and convert rpm to cycles/second. Our goal is to find the frequency, in Hertz, and the angular frequency of the simple harmonic motion.

DEVELOP Convert revolutions per minute (rpms) to frequency (revolutions per second). For part (b) convert the frequency to an angular frequcy using $f = \omega/(2\pi)$.

EVALUATE

(a) $f = (600 \text{ rpm})(1.00 \text{ min}/60.0 \text{ s}) = 10.0 \text{ s}^{-1}$.

(b) The angular frequency is w $= 2\pi f = (2\pi \text{ rad})(10.0 \text{ s}^{-1}) = 62.8$ rad/s.

ASSESS Circular motion and simple harmonic motion are very closely related! That's why we use the symbol ω for both—it's the same in both.

36. **INTERPRET** You want to compare the vibrational energy to the kinetic energy of your friend's car.

DEVELOP The car's kinetic energy is $K = \frac{1}{2}mv^2$, while the total energy in the oscillation is

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}m\omega^2 A^2 \tfrac{1}{2}m(2\pi fA)^2$$

We're told to assume the mass here is the whole car, even though the wheels and other parts of the car do not oscillate with the same magnitude or frequency.

EVALUATE The fraction of energy in oscillatory motion vs. kinetic energy is

$$\frac{E}{K} = \left(\frac{2\pi fA}{v}\right)^2 = \left(\frac{2\pi(0.67 \text{ Hz})(0.18 \text{ m})}{(20 \text{ m/s})}\right)^2 = 0.14\%$$

ASSESS This is a very small percentage, so your friend shouldn't worry that the oscillations are affecting his/her fuel efficiency. It might seem strange to even consider the effect of a car's vibrations on fuel consumption, since the engine is not directly driving this up and down motion. But the oscillations are a consequence of the forward motion: the car's wheels hit a bump, and some of the engine's work is "lost" into vibrations. It's not all that different from an inelastic collision, in which some of the initial kinetic energy of the incoming particles is lost to internal vibrations that manifest themselves as heat or sound.

37. **INTERPRET** The problem is about damped harmonic motion. We are interested in finding out how long it takes for the vibration amplitude of a piano string to drop to half its initial value.

DEVELOP The solution to the damped harmonic motion is given by Equation 13.17:

$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi)$$

Thus, the amplitude is half the initial value when $e^{-bt/2m} = 0.5$.

EVALUATE Taking the natural logarithms of both sides of the equation above gives $bt/2m = \ln 2$. Thus, the time for the amplitude to be halved is

$$t = \frac{2m}{b}\ln 2 = \frac{\ln 2}{2.8 \text{ s}^{-1}} = 0.25 \text{ s}$$

ASSESS Since the amplitude drops by half in just 0.25 s, we conclude that the damping must be rather strong.

40. **INTERPRET** This problem involves the simple harmonic motion of the oxygen atoms in a carbon-dioxide molecule. We are to interpret the chemical bonds as springs are given the frequency of oscillation of the oxygen atoms and their mass, from which we can calculate the spring constant.

DEVELOP The relationship between the oscillating frequency and the spring constant is given by Equation 13.7a, $\omega = \sqrt{k/m}$. Use the conversion factor in Appendix C to convert the mass of the oxygen atom from u to kg.

EVALUATE Solving for k and inserting the given quantities gives

$$\omega = 2\pi f = \sqrt{k/m}$$
$$k = (2\pi f)^2 m$$
$$= (2\pi \times 4.0 \times 10^{13}\ \text{Hz})^2 (16 \times 1.66 \times 10^{-27}\ \text{kg}) = 1.7 \times 10^3\ \text{N/m.}$$

ASSESS Note that we converted from angular frequency to frequency because the data is given in frequency.

42. **INTERPRET** This problems asks for the resonant frequency of the human eye if we model it as a mass and the muscles that hold it as a spring.

DEVELOP The natural (or resonant) frequency is $\omega_0 = \sqrt{k/m}$.

EVALUATE In terms of oscillations per second, the resonant frequency of the human eye is

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2.5\ \text{kN/m}}{7.5\ \text{g}}} = 92\ \text{Hz}$$

ASSESS You wouldn't be able to shake your head this fast by yourself, but you might feel this much vibration in an extremely bumpy amusement ride, for example.

47. **INTERPRET** We will analyze a bacteria protein that undergoes simple harmonic motion.

DEVELOP The peak force occurs when the mass-spring system is maximally displaced from equilibrium: $F_{\text{peak}} = k|x_{\text{max}}|$. The maximum displacement for simple harmonic motion is by definition the amplitude: $|x_{\text{max}}| = A$. From this we can find the spring constant, and from Equation 13.7b, the effective mass is $m = k/(2\pi f)^2$.

EVALUATE (a) The spring constant of the dynein-microtubule system is

$$k = \frac{F_{\text{peak}}}{A} = \frac{1.0\ \text{pN}}{15\ \text{nm}} = 0.0667\ \text{pN/nm} = 67\ \mu\text{N/m}$$

(b) Given the frequency of oscillation, the effective mass being oscillated is

$$m = \frac{k}{(2\pi f)^2} = \frac{66.7\ \mu\text{N/m}}{(2\pi \cdot 70\ \text{Hz})^2} = 3.4\times 10^{-10}\,\text{kg}$$

ASSESS The effective mass is about 100 times that of a typical human cell. This seemingly large value may be because the system is moving through a viscous fluid that acts like additional mass.

56. **INTERPRET** This problem involves conservation of motion and Newton's second law ($F = ma$). To apply conservation of energy, we need to choose a zero for the potential energy, so we will choose Jane's starting position to have zero potential energy. Apply Newton's second law to find the maximum tension in the vine.

DEVELOP To find the maximum tensile force, apply Newton's second law to Jane as she is swinging. First, make a free-body diagram of her (see figure below, left side). Because Jane is executing circular motion, here acceleration is centripetal and is directed toward the center of the circle (i.e., along the vine). It is given by $a_c = v^2/R$ (see Equation 3.16). Using this result, Newton's second law gives

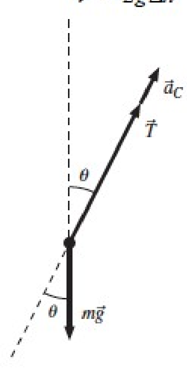
$$F_{\text{net}} = ma$$
$$T - mg\cos\theta = m\frac{v^2}{R}$$
$$T = m\frac{v^2}{R} + mg\cos\theta$$

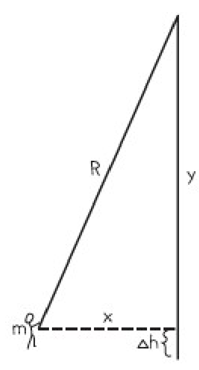
Thus, we see that the maximum tension occurs when $\theta = 0$, which is when Jane is at the bottom of her trajectory. To find the value of this maximum tension, we need to calculate her speed at that point, which we can do using conservation of energy. The initial distance between Tarzan and Jane in Figure 13.12 is $d = 8$ m. Jane's mass is $m = 60$ kg. Draw a sketch of the situation that shows how far down Jane will swing (see figure below, right side). The distance $x = d/2$, and we can use the Pythagorean theorem to find y and thus Δh. This gives

$$R^2 = x^2 + y^2 = x^2 + (R - \Delta h)^2$$
$$\Delta h = R - \sqrt{R^2 - x^2}$$

Because we chose her starting position to have zero potential energy, her initial total energy (potential plus kinetic) is zero. By conservation of total mechanical energy, her final total energy at the bottom of the swing must also be zero, so

$$0 = K_f + U_f = \frac{1}{2}mv^2 - mg\Delta h$$
$$v^2 = 2g\Delta h$$





EVALUATE The maximum tension in the vine occurs at $\theta = 0$, so

$$T_{max} = m\frac{v^2}{R} + mg = m\frac{2g\Delta h}{R} + mg = mg\left[\frac{2}{R}\left(R - \sqrt{R^2 - (d/2)^2}\right) + 1\right] = mg\left(3 - \sqrt{4 - (d/R)^2}\right)$$

$$= (60\text{ kg})(9.8\text{ m/s}^2)\left[3 - \sqrt{4 - \left(\frac{8.0\text{ m}}{25\text{ m}}\right)^2}\right] = 600\text{ N}$$

Thus, the maximum tension is 600 N (to two significant figures) and it occurs at the bottom of Jane's trajectory.

Assess This force is only slightly greater than Jane's weight, since she is moving rather slowly. We also see that $d \le 2R$ for the radical to remain real, which makes sense because $2R$ is the maximum length she can travel horizontally. This extremum will give the maximum T_{max} value, which is $3mg$.

62. **Interpret** In this problem we want to recover the Newton's second law for simple harmonic motion by differentiating the expression for total mechanical energy.

Develop Newton's second law applied to the mass-spring system gives $m\,d^2x/dt^2 = -kx$ (Equation 13.3). To recover this expression from the energy equation, we shall make use of $v = dx/dt$ and $dv/dt = d^2x/dt^2$.

Evaluate Since E is a constant, we have

$$\frac{dE}{dt} = 0 = \frac{d}{dt}\left(\frac{1}{2}kx^2 + \frac{1}{2}mv^2\right) = kx\frac{dx}{dt} + mv\frac{dv}{dt} = kxv + mv\frac{dv}{dt}$$

$$= v\left(kx + m\frac{dv}{dt}\right)$$

$$= v\left(kx + m\frac{d^2x}{dt^2}\right)$$

The above expression implies that the term in brackets is zero, or $m\,d^2x/dt^2 = -kx,$ which is Equation 13.3.

Assess Because there is no damping term in the equation of motion, the total energy in the system is conserved.