Solutions

Physics 6A Midterm II – 12:30 pm lecture Quarter: Fall 2011 Instructor: Sue Carter

You are in a cave and the Fog brother just hit a vampire bat with a ball. That bat seems to be knocked out, but another, even larger bat with impressive looking fangs wakes up.

1a -- 12 pts) The Fog brother uses his rubber sling like a rubber band to fire the ball straight up at this larger bat. If the 100 gram ball is shot vertically from the sling (spring-constant of 20 N/m) and the ball has to reach a height of 10 meters to hit the bat, how far back does the Fog brother have to stretch the sling prior to release?

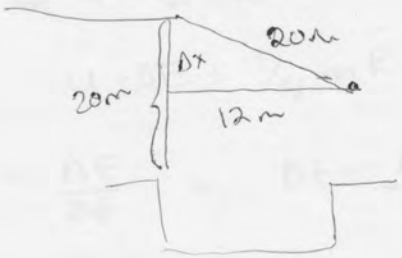
$\Delta U_{sp} = \Delta U_g$

$\frac{1}{2} k x^2 = mgh$

$x^2 = \frac{2mgh}{k}$ $\quad x = \sqrt{2mgh/k}$

$x = \sqrt{\frac{0.1\,kg \cdot 9.8\,m/s^2 \cdot 10\,m \cdot 2}{20\,N/m}} = 0.99\,m$

1b – 13 pts) While successful, now all the bats are awake and chasing you. The Fog brothers show you an alternative route to exit the cave which involves crossing a gorge. A 20-m long vine hangs vertically from the ceiling on one side of a 12-m wide gorge. You run up, hoping to grab the vine, swing over the gorge, and drop vertically off the vine to land on the other side. How fast must you run?



$\Delta x = \sqrt{(20m)^2 - (12m)^2} = 16m$

$h = 20\,m - \Delta x$

$h = 4\,m$

$\frac{1}{2} m v^2 = mgh$

$v^2 = 2gh$

$v = \sqrt{2gh}$

$v = \sqrt{2 \cdot 9.8\,m/s^2 \cdot 4m}$

$= 8.85\,m/s$

2a - 13 pts) You jump on your bike and race home, but now you see a suspicious black motorcycle heading at a speed of 5 m/s directly towards you. The Fog brother stops, takes out his sling, and releases a rubber ball at 20 m/s directly towards the motorcycle. The elastic rubber ball bounces off the motorcycles windshield. At what speed does the ball come flying back at you (assume the motorcycle's mass is much greater than the ball's mass)?

Elastic Collision

$$v_{1f} = \frac{2m_1}{m_1+m_2} v_{1i} + \frac{m_2-m_1}{m_1+m_2} v_{2i}$$

Let $m_1 >> m_2$ then

$$v_{2f} = \frac{2m_1}{m_1} v_{1i} - \frac{m_1}{m_1} v_{2i}$$

$$= 2v_{1i} - v_{2i}$$

$$= 2(5\,m/s) - (-20\,m/s) = \boxed{30\,m/s}$$

Or $v_{rf} = v_{1i} - v_{2i} + v_{1f}$

$$= 5\,m/s - (-20\,m/s) + 5\,m/s = \boxed{30\,m/s}$$

2b - 12 pts) Luckily, the motorcycle runs out of gas and you get home safely. However, shortly afterward, your house is surrounded by the same man (the Fog brothers call him Dave) on a motorcycle along with his friends. Apparently Dave's motorcycle utilizes a fly wheel for back up power. The fly wheel consists of a 30 cm diameter disk with mass of 20 kg and rotates at 10,000 rpm. If the motorcycle requires 1000 Watts to run, for how long will the fly wheel power the motorcycle?

$K_{rot} = U = \frac{1}{2} I \omega^2$ $\quad I = \frac{1}{2} m R^2$ $\quad \omega = 10{,}000$ rpm $= 1050$ rad/s

so $U = \Delta E = \frac{1}{4} m R^2 \omega^2$

$$P = \frac{\Delta E}{\Delta t} \quad \text{so} \quad \Delta t = \frac{\Delta E}{P} = \frac{\frac{1}{4} m R^2 \omega^2}{P}$$

$$\Delta t = \frac{\frac{1}{4} \cdot 20\,kg\,(0.15\,m)^2 \cdot (1050\,rad/s)^2}{1000\,W}$$

$$= 124\,s \approx 2\,min$$

3a - 13 pts) Dave decides to sit idle on his motorcycle, taunting you. You are concerned when pulls out the fly wheel (30 cm diameter disk with mass of 20 kg and rotating at 10,000 rpm) and attaches a small blade (mass=1 kg) to the edge. What is the velocity (in m/s) of the blade tip (treat the blade as an added point mass at the disk's edge)?

$I_i = \frac{1}{2} m_{FW} r^2$

$I_f = \frac{1}{2} m_{FW} r^2 + m_B r^2$

$I_i \omega_i = I_f \omega_f$

$I_i = \frac{1}{2} \cdot 20\,kg \cdot (0.15\,m)^2 = 0.225\,kg\,m^2$

$I_f = \frac{1}{2} \cdot 20\,kg \cdot (0.15\,m)^2 + 1\,kg \cdot (0.15\,cm)^2$

$= 0.247\,kg\,m^2$

$\omega_f = \omega_i \frac{I_i}{I_f} = 10{,}000\,rpm \cdot \frac{0.225\,kg\,m^2}{0.2475\,kg\,m^2} = 9090\,rpm$

$V = \omega r = 9090\,rpm \times \frac{2\pi}{1\,rev} \times \frac{1\,min}{60\,sec} \times 0.15\,m$

$= 143\,m/s$

3b -- 12 pts) Your brother comes home and the motorcycles mysteriously disappear. You are very happy to see your older brother, but are surprised when your dog suddenly attacks him. You pick up your nerf gun and fire rubber bullets at your dog to slow down his attack. If each bullet has a mass of 10 grams and travels at 20 m/s, what total impulse do you deliver to your dog if you fire 5 bullets per second for 5 seconds.

$J = N_{bullets} \cdot m_B \cdot v_B$

$= 5\,bullets/sec \times 5\,sec \times 0.01\,kg \cdot 20\,m/s$

$= 5\,Ns$

Luckily that seems to stop your dog, but you start getting worried when you can't see your brothers reflection in the hallway mirror. You call the Fog brothers for help.

4a-13 pts) The Fog brothers tell you that the vampires are taking over, and that they have to fire a suborbital rocket to warn the vampire-hunters across the country. If the rocket follows a up-down vertical path, how much energy does it take to launch a 10-kg rocket up a vertical trajectory that peaks at 1500 km altitude?

$$\Delta U = \frac{GM_e m_r}{R_e + h} - \frac{GM_e m_r}{R_e}$$

$$= 6.67\times10^{-11}\ Nm^2/kg^2 \cdot 5.97\times10^{24}\ kg \cdot 10\ kg \cdot \left[\frac{1}{6.37\times10^{6}\ m} - \frac{1}{1.787\times10^{6}\ m}\right]$$

$$= 1.2\times10^{8}\ J$$

Note: $\Delta U = mgh$ -6

-1 $R_e + h$

$+1$ R_e

4b – 12 pts) The Fog brothers join you, your mom, and your Mom's friend Mack for dinner at your house. You can cook dinner using the stove top or microwave. The instructions tell you to cook the rice stew for 45 minutes at low heat on the stove top or for 15 minutes under high heat in the microwave. A burner on the stove top takes 1600 Watts and is on 30% of the time at low heat. The microwave takes 1000 Watts and is on 100% of the time at high power. If the cost of energy is \$0.20 per kW-hr, what appliance should you use and how much money will you save?

Cost of stove: $0.3\times1600\ W\times0.75\ hr\times \$0.20/kW\text{-}hr\times\frac{1\ kW}{1000\ W} = \$0.072$

Cost of microwave: $1\times1000\ W\times0.25\ hr\times \$0.20/kW\text{-}hr\times\frac{1\ kW}{1000\ W} = \$0.050$

Use microwave and save $\$0.07 - \$0.05 = \$0.02$

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