Dave invites you to a party with his friends in the old hotel, and you drink a strange red fluid.

2a - 13 pts) You later find yourself alone laying down on a merry-go-round-like contraption, shaped like a solid disk, that has a radius of 5 meters and an average mass (including you) of 300 kg. It is being pushed from rest by one of Dave's friends who applies a constant 50 N normal force. Assuming the mass is evenly distributed over the disk, how long does it take for the merry-go-round to reach a centripetal acceleration of 10 m/s².

$$a_c = 10\ m/s^2 = \frac{v^2}{r} = \omega^2 r$$

$$\omega = \sqrt{\frac{10\ m/s^2}{5\ m}} = \sqrt{2}\ rad/s = 1.414\ rad/s$$

$$\tau = I\alpha = rF \qquad I = \tfrac{1}{2}mr^2 \text{ for solid disk}$$

$$\text{So} \quad \alpha = \frac{rF}{\frac{1}{2}mr^2} = \frac{2F}{mr} = \frac{2 \cdot 50\ N}{300\ kg \cdot 5\ m} = 0.066\ rad/s^2$$

$$\Delta t = \frac{\omega}{\alpha} = \frac{\sqrt{2}\ rad/s}{0.066\ rad/s^2} = \boxed{21\ s}$$

2b - 12 pts) After the merry-go-round reaches an angular velocity of 100 rpm, Dave's friend (mass = 70 kg) jumps onto the outside rim of the merry-go-round. What is the resulting angular velocity of the merry-go-round (including you and Dave's friend)?

Note: -4 for using $\frac{1}{2} m_{DF} r^2$

$$I_0 \omega_0 = I_f \omega_f$$

$$I_0 = \tfrac{1}{2} m_{MGR} r^2 \qquad I_f = \tfrac{1}{2} m_{MGR} r^2 + m_{DF} r^2$$

$$\omega_f = \frac{I_0}{I_f}\omega_0 = \frac{\frac{1}{2} m_{MGR} r^2}{\frac{1}{2} m_{MGR} r^2 + m_{DF} r^2} \cdot \omega_0$$

$$= \frac{\frac{1}{2} 300\ kg\ (5\ m)^2}{\frac{1}{2} \cdot 300\ kg \cdot (5\ m)^2 + 70\ kg \cdot (5\ m)^2} \cdot 100\ rpm$$

$$= \boxed{68\ rpm} \quad \text{or} \quad \cancel{10.8}\ 7.14\ rad/s$$